Transport Coefficients in a Strongly Coupled Baryon-Rich QGP

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Mostly based on:

arXiv:1311.6675
arXiv:1412.2968
arXiv:1505.07894
arXiv:1507.06972

BNL Workshop “Opportunities for Exploring Longitudinal Dynamics in Heavy Ion Collisions at RHIC”, Jan. 2016

OUTLINE

- Perfect fluidity, strongly coupled QGP, black hole engineering

- Non-conformal holographic calculations of transport coefficients at zero baryon chemical potential

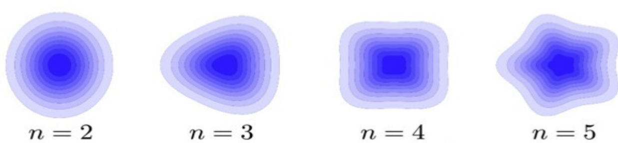
- Transport coefficients for a baryon rich QGP and the critical endpoint

- Final remarks

Perfect fluidity: an emerging property of QCD

Behavior consistent with a strongly interacting fluid

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left[1 + \sum_n 2v_n \cos\left[n\left(\phi - \psi_n\right)\right]\right]$$

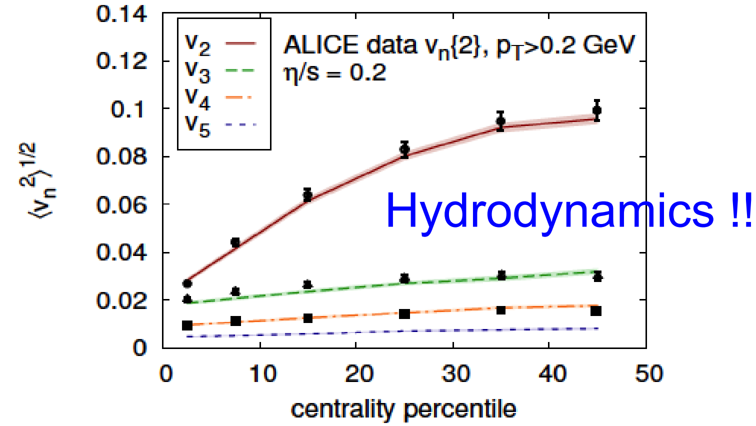


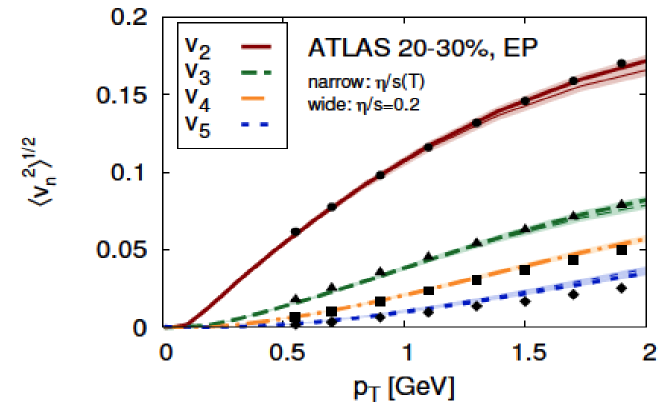
QGP as a "little big bang"

QGP seems to be a strongly coupled relativistic fluid

$$\eta/s \sim 0.1 - 0.2$$

2.76 TeV, Pb+Pb at LHC

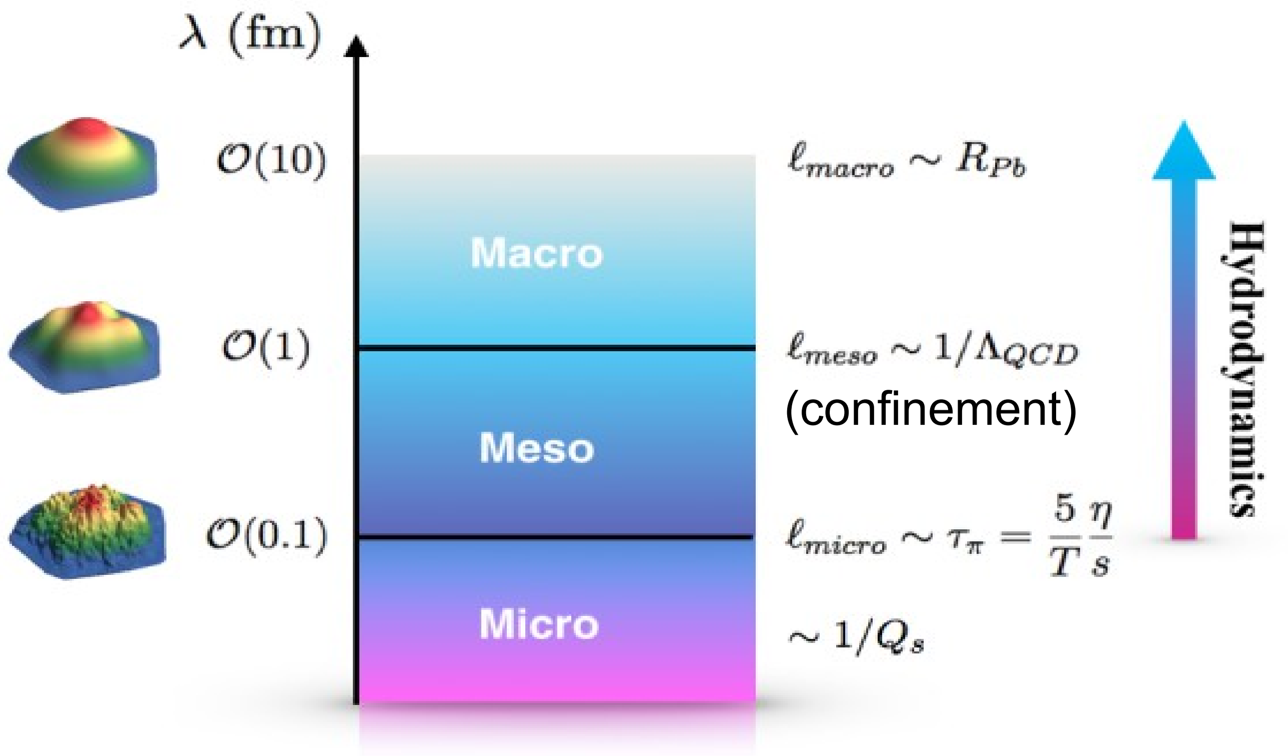




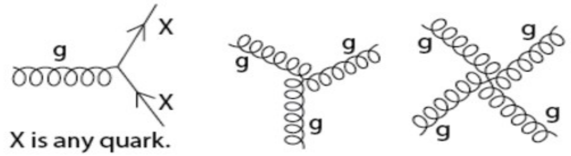
Gale et al, PRL 110, 012302 (2013)

A tale of scales in relativistic heavy ion collisions

J. Noronha-Hostler, JN, M. Gyulassy, PRC (2016) arXiv:1508.02455



Perfect fluidity very hard to obtain from



Strong coupling phenomenon?

Challenge:

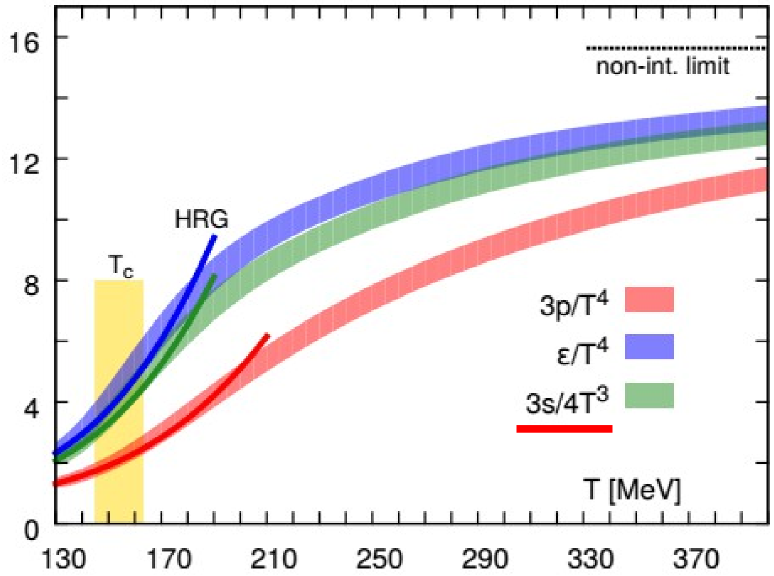
Compute η/s in the range of temperatures T ~ 150 - 400 MeV probed in heavy ion collisions ("perfect fluidity").

This is not that hard since 50% of it has been already solved !!!!!

The entropy density has been already computed non-perturbatively on the lattice

Solid QFT work
+ computers

HotQCD (2014)



The real problem of (2) now is the numerator ...

Holy Grail

Retarded energy-momentum tensor correlator

$$G_R^{\mu\nu\alpha\beta}(x-x') = i\,\theta(x-x')\langle[\hat{T}^{\mu\nu}(x), \hat{T}^{\alpha\beta}(x')]\rangle_T$$

Thermal QCD state

Kubo formula

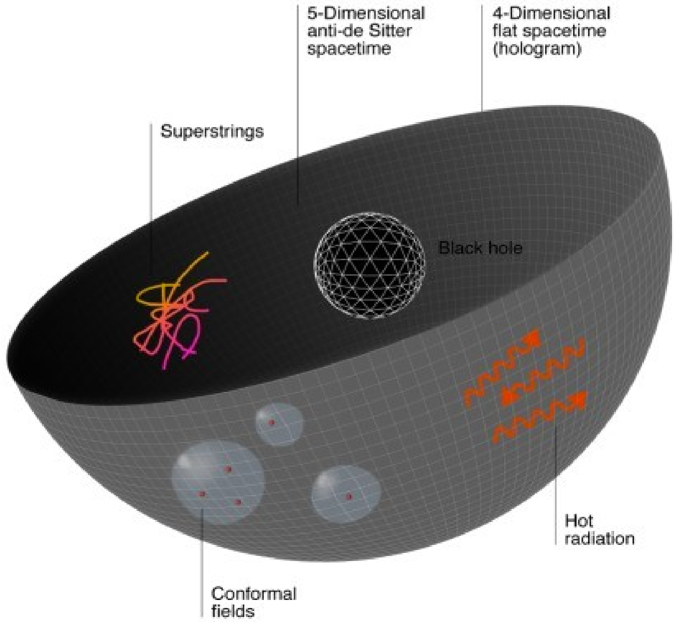
$$\boxed{\eta = i\,\partial_\omega G_R^{xyxy}(\omega, \mathbf{0})\Big|_{\omega=0}}$$

- Cannot be computed directly on the lattice.

- No one currently knows how to compute this in QCD in its full glory.

Holography (gauge/string duality)

Maldacena 1997; Witten 1998; Gubser, Polyakov, Klebanov 1998



Strong coupling limit of QFT in 4 dimensions

⇕

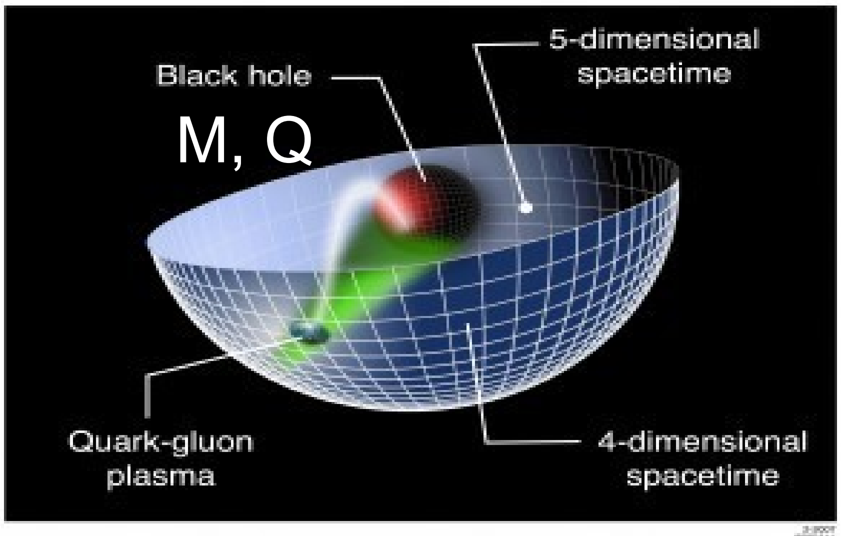
String Theory/Classical gravity in d>4 dimensions

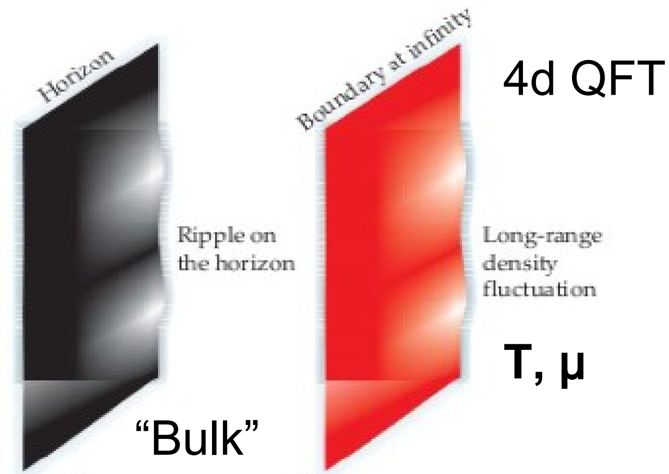
<u>Universality and perfect fluidity</u>

"Any strongly interacting quantum many-body system at finite density and temperature with sufficiently many d.o.f / volume is predicted to behave at low energies as a perfect fluid"

The holographic correspondence at finite temperature and density

Near-equilibrium fluctuations in the plasma ~ black brane fluctuations !!!!



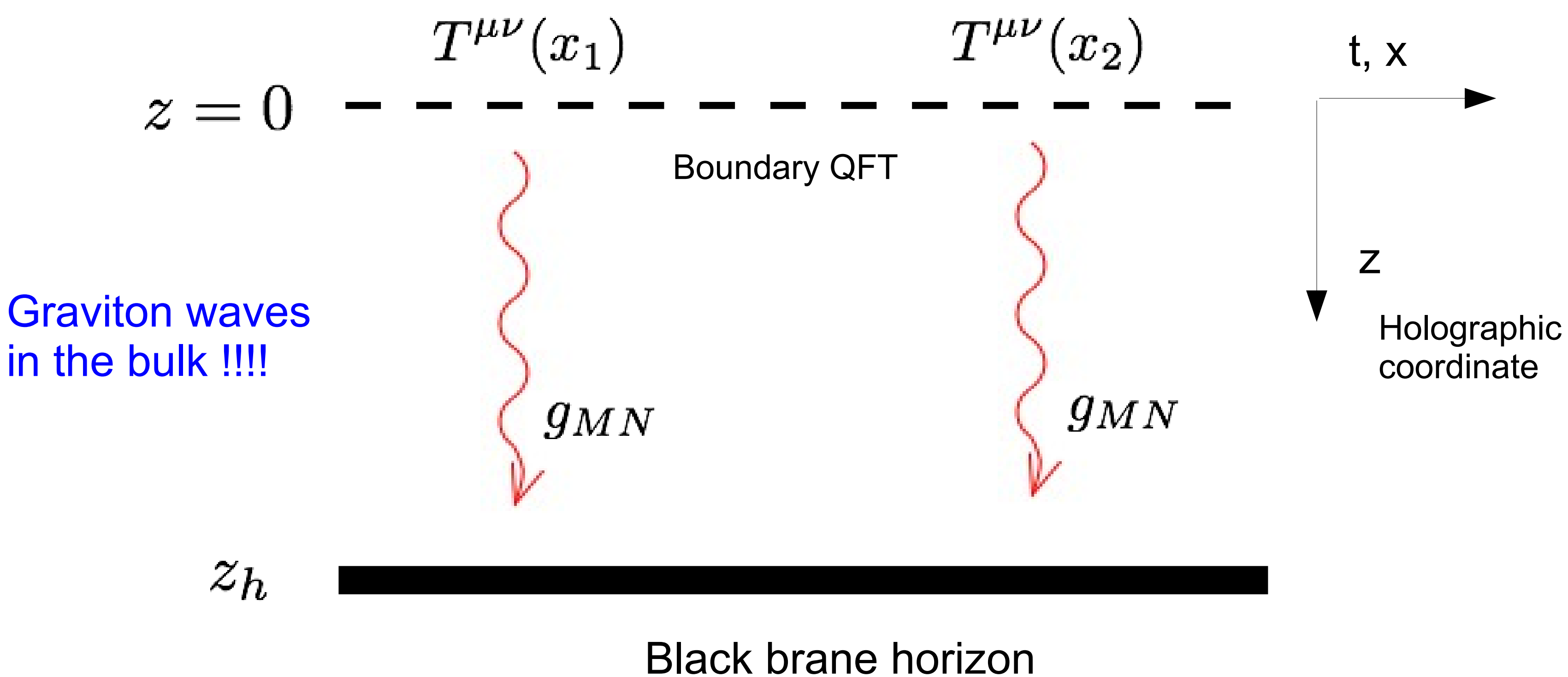


Fluid dynamics from black hole physics

Quasiparticle dynamics replaced by geometry

- Why is this useful for QGP physics?

Retarded correlator of the energy-momentum tensor $G_R^{xy,xy}$



Universality and perfect fluidity

$\lambda \gg 1$ in QFT $\rightarrow$ string theory in weakly curved backgrounds

d.o.f. / vol. $\rightarrow \infty$ in QFT $\rightarrow$ vanishing string coupling

T, μ in QFT $\rightarrow$ spatially isotropic black brane

For anisotropic models
there is violation
see arXiv:1406.6019

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

Universality of shear viscosity

Kovtun,Son,Starinets, 2005

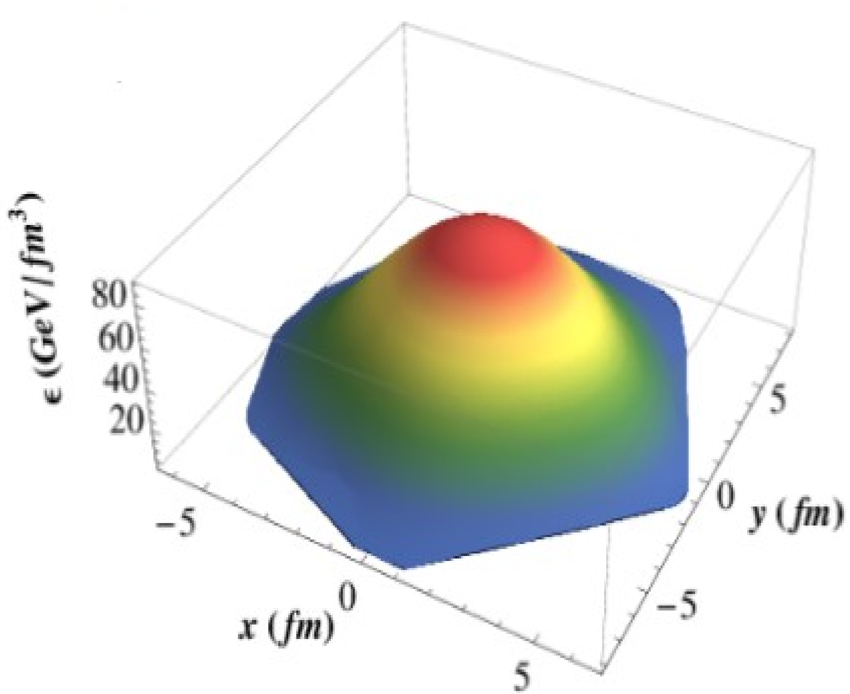
Universality of black hole horizons $\Rightarrow$ **HOLOGRAPHY** $\Rightarrow$ Universality of transport coefficient in QFT

Perfect fluidity is a prediction of holography

This was all we needed < 2010. The QGP was modeled to be

Smooth over scales of the order ~ 5 -10 fm

Conformal dynamics, $\varepsilon = 3P$



macro $\partial\varepsilon/\varepsilon_0 \sim 1/L$

micro $\ell \sim 1/T \sim 1/\Lambda_{QCD}$

Knudsen number

$$\boxed{K_N \sim \ell\,\partial\varepsilon < 0.1}$$

Fluid dynamics at scales of the size of a large nucleus

Reasonable separation of scales

$$\boxed{K_N \sim \ell\,\partial\varepsilon < 0.1}$$

QGP as a relativistic dissipative fluid

$$\nabla_\mu T^{\mu\nu} = 0$$

conservation law

$$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu + P\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Inviscid part

Dissipative part

Relativistic Navier-Stokes: $\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \mathcal{O}(\partial^2\varepsilon, \partial^2 u)$

assumed to be small

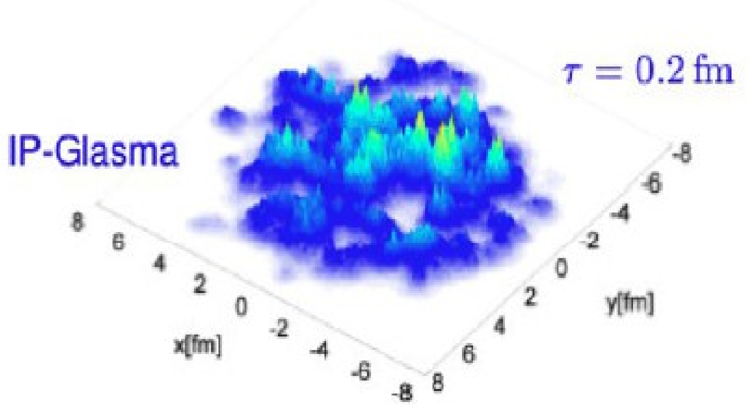
Shear tensor

$$\sigma_{\mu\nu} = 2\Delta^{\alpha\beta}_{\mu\nu}\nabla_\alpha u_\beta$$

Flow velocity

$$u_\mu u^\mu = -1$$

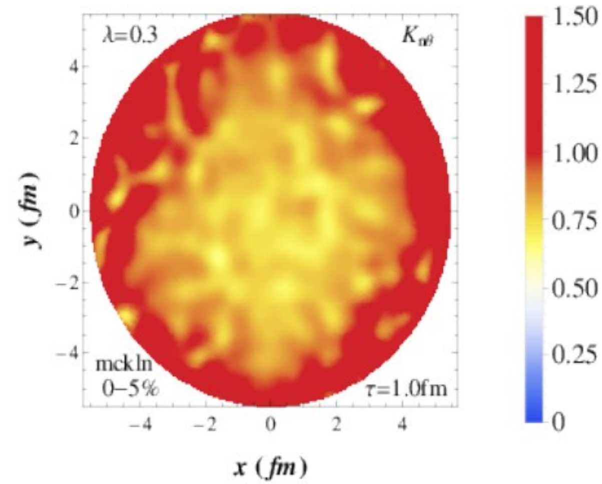
Nowadays, current prejudice about QGP



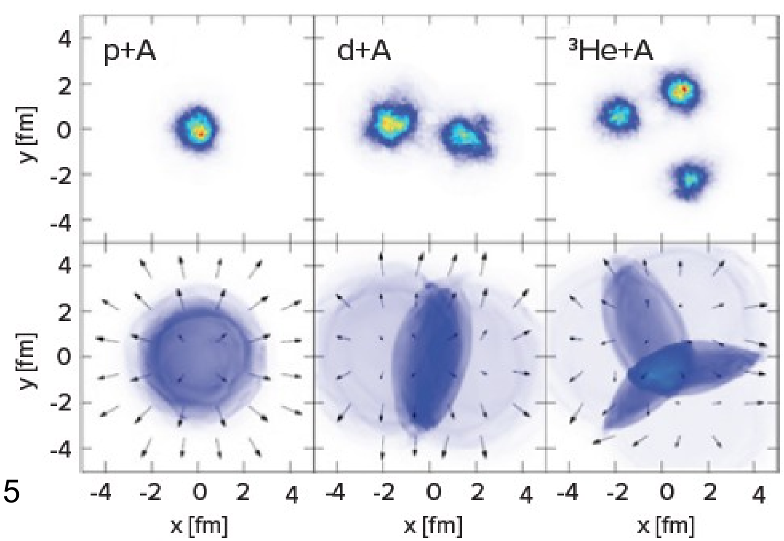
Schenke, Tribedy, Venugopalan, 2012

J. Noronha-Hostler, JN, M. Gyulassy, arXiv:1508.02455

Knudsen number (MCKLN)



Indication of hydrodynamic behavior now at even short scales ???



Schenke, 2015

macro $\partial\varepsilon/\varepsilon_0 \sim \Lambda_{QCD}$

microscopic scale???

Non-conformal relativistic hydrodynamics at 2nd order in gradients

2nd order gradient expansion for a non-conformal QGP in <u>curved spacetime</u>

S. Finazzo, R. Rougemont, H. Marrochio, JN, JHEP 1502 (2015) 051

Shear channel:

$$\tau_\pi\left(D\pi^{\langle\mu\nu\rangle}+\frac{4\theta}{3}\pi^{\mu\nu}\right)+\pi^{\mu\nu}=-\eta\sigma^{\mu\nu}+\kappa\left(\mathcal{R}^{\langle\mu\nu\rangle}-2u_\alpha u_\beta\mathcal{R}^{\alpha\langle\mu\nu\rangle\beta}\right)+\tau_\pi\,\pi^{\mu\nu}\;D\ln\left(\frac{\eta}{s}\right)$$
$$+\frac{\lambda_1}{\eta^2}\pi_\lambda^{\langle\mu}\pi^{\nu\rangle\lambda}-\frac{\lambda_2}{\eta}\pi_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda}-\lambda_3\Omega_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda}+2\kappa^*\,u_\alpha u_\beta\mathcal{R}^{\alpha\langle\mu\nu\rangle\beta}$$
$$+\tau_\pi^*\pi^{\mu\nu}\,\frac{\Pi}{3\zeta}+\lambda_4\nabla^{\langle\mu}\ln s\,\nabla^{\nu\rangle}\ln s$$

Bulk channel:

$$\tau_\Pi\left(D\Pi+\Pi\theta\right)+\Pi=-\zeta\theta+\frac{\xi_1}{\eta^2}\pi_{\mu\nu}\pi^{\mu\nu}+\frac{\xi_2}{\zeta^2}\Pi^2+\tau_\Pi\,\Pi\;D\ln\left(\frac{\zeta}{s}\right)$$
$$+\xi_3\Omega_{\mu\nu}\Omega^{\mu\nu}+\xi_4\nabla_\mu^\perp\ln s\,\nabla_\perp^\mu\ln s+\xi_5\mathcal{R}+\xi_6u^\mu u^\nu\mathcal{R}_{\mu\nu}$$

17 temperature dependent transport coefficients!!

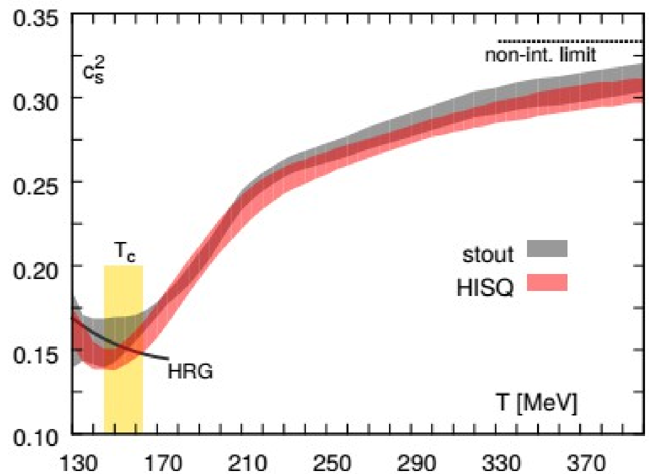
Another important point ...

QGP is not conformal → bulk viscosity must be taken into account

$$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu + P\Delta^{\mu\nu} + \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi$$

Denicol et al., PRC 2009.
Monnai and Hirano, PRC 2009.
P. Bozek, PRC 2010.
Dusling and Schafer, PRC 2012.
J. Noronha-Hostler, JN, et. al., PRC 2013, 2014.
S. Ryu et al., PRL 2015.

Speed of sound



Realistic holographic modeling of the QGP must incorporate violation of conformal invariance

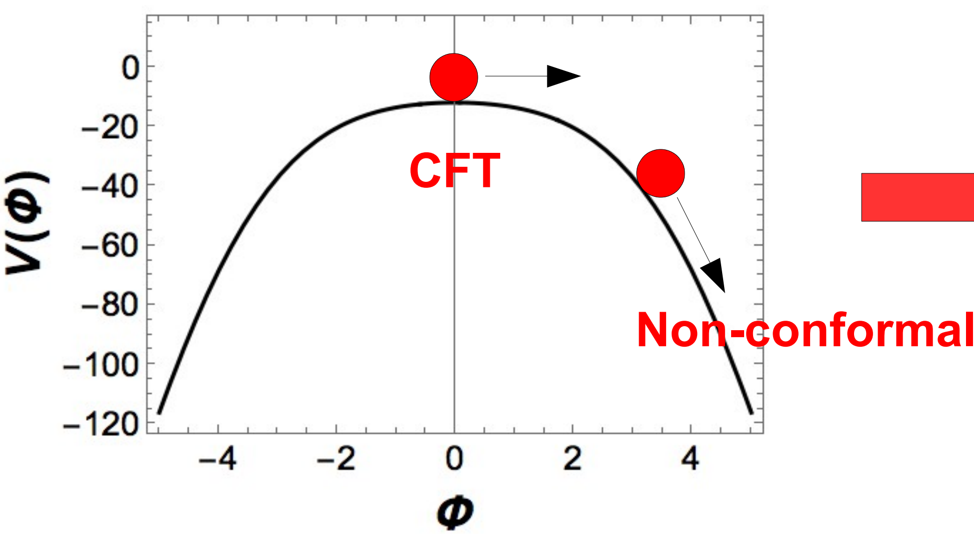
Black hole engineering and the non-conformal QGP

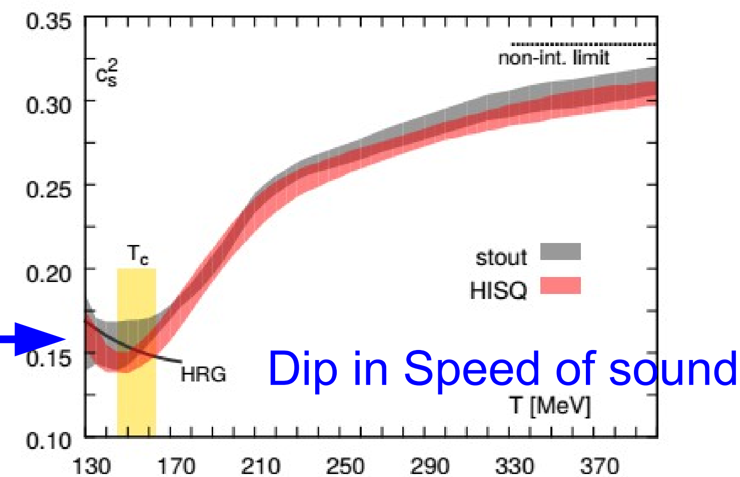
Minimal 5d bulk holography for a non-conformal plasma

Gubser et al. 2008
Kiritsis et al, 2008
Noronha, 2009

$$S_{\rm ES}^{(\rm bulk)} = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R - \frac{(\partial_M\Phi)^2}{2} - V(\Phi)\right]$$

Φ is the scalar field and $V(\Phi)$ is the scalar potential





- Start with a nontrivial UV fixed point – strongly interacting CFT.

- Add a relevant scalar operator → nontrivial IR behavior

- The scalar potential is an **input** of the theory (black hole engineering)

$$V(\Phi) = \frac{-12\cosh\gamma\Phi + b_2\Phi^2 + b_4\Phi^4 + b_6\Phi^6}{L^2}$$

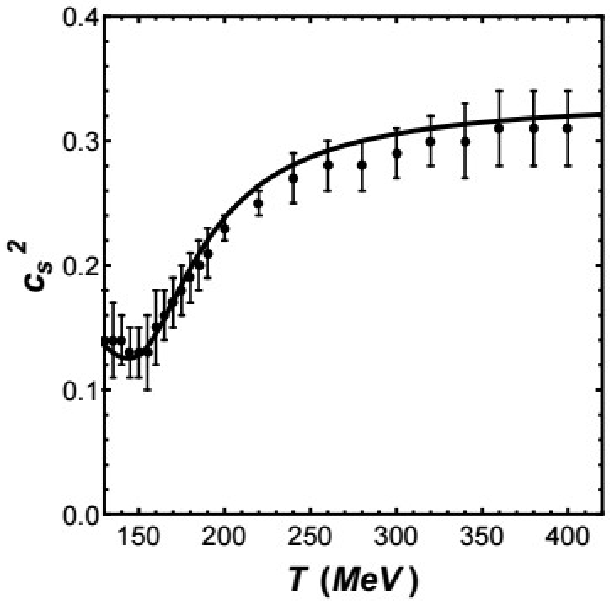
$\gamma = 0.606,\ b_2 = 0.703,\ b_4 = -0.1,\ b_6 = 0.0034$ $\Delta = 3$

completely fixed by requiring that the model describes lattice QCD data at finite T (and zero baryon density)

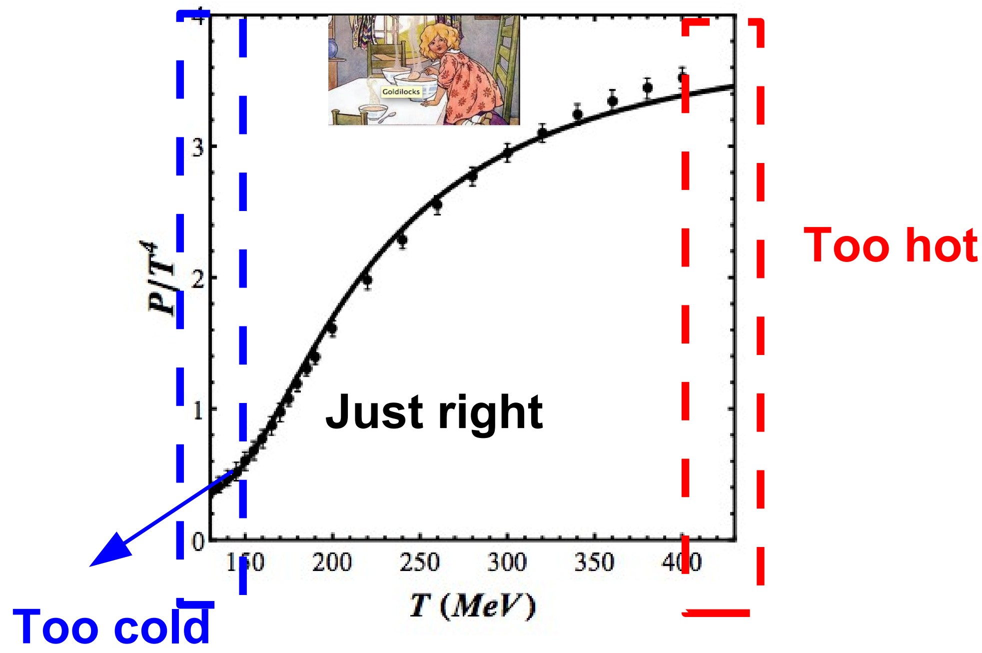
Holographic description of QGP thermodynamics

S. Finazzo, R. Rougemont, H. Marrochio, JN, JHEP 1502 (2015) 051

Lattice data from Borsanyi et al, JHEP 08 (2012) 053.



"Holographic Goldilocks"



5d bulk metric (Gubser gauge)

$$ds^2 = e^{2A(\Phi)}\left(-h(\Phi)dt^2 + dx_i^2\right) + e^{2B(\Phi)}\frac{d\Phi^2}{h(\Phi)}$$

Bulk viscosity

S. Finazzo, R. Rougemont, H. Marrochio, JN, JHEP 1502 (2015) 051

The Kubo formula is $\zeta = -\frac{4}{9} \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im}\left[G_R(\omega, \vec{q} = \vec{0})\right]$

Retarded correlator

$$G_R(\omega, \vec{q}) \equiv -i \int_{\mathbb{R}^{1,3}} d^4x \, e^{i(\omega t - \vec{q}\cdot\vec{x})} \theta(t) \left\langle \left[\frac{1}{2} T_a^a(t, \vec{x}), \frac{1}{2} T_b^b(0, \vec{0})\right] \right\rangle$$

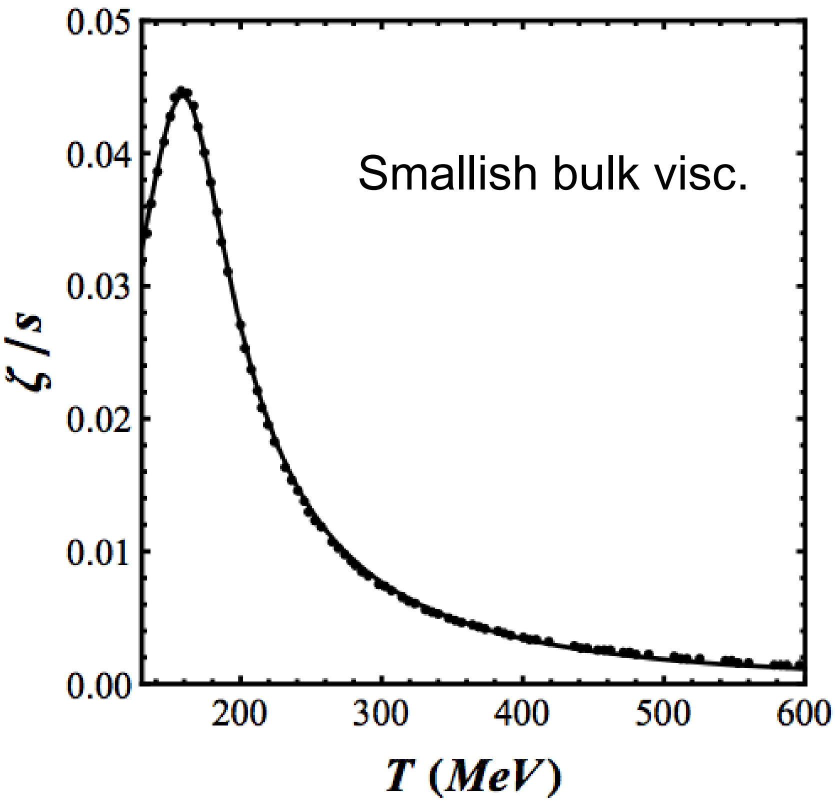
Infalling b.c. for metric fluctuations $\psi \equiv h_x^x = e^{-2A(\phi)} h_{xx}$

$$\psi'' + \left(\frac{1}{3A'} + 4A' - 3B' + \frac{h'}{h}\right)\psi' + \left(\frac{e^{-2A+2B}}{h^2}\omega^2 - \frac{h'}{6hA'} + \frac{h'B'}{h}\right)\psi = 0,$$

Bulk viscosity

S. Finazzo, R. Rougemont, H. Marrochio, JN, JHEP 1502 (2015) 051

Infalling boundary conditions: $\psi(\phi \to \phi_H) \approx C e^{i\omega t} |\phi - \phi_H|^{-\frac{i\omega}{4\pi T}}$



General formula Gubser et al, 2009

$$\frac{\zeta}{s} = \frac{\eta}{s} |C|^2 \frac{V'(\phi_H)^2}{V(\phi_H)^2}$$

Parametrization for hydro

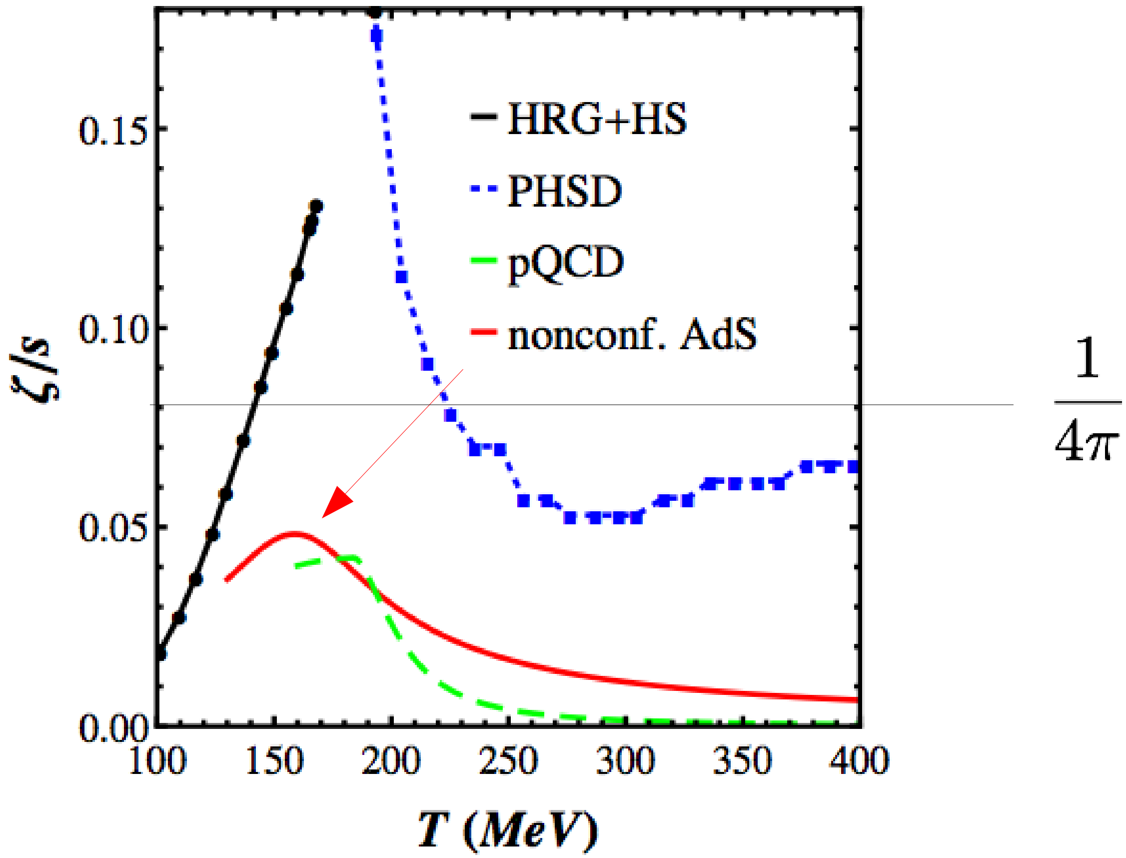
$$\frac{\zeta}{s}\left(x = \frac{T}{T_c}\right) = \frac{a}{\sqrt{(x-b)^2 + c^2}} + \frac{d}{x^2 + e^2}$$

$T_c = 143.8$ MeV

a	b	c	d	e
0.01162	1.104	0.2387	-0.1081	4.870

Bulk viscosity

S. Finazzo, R. Rougemont, H. Marrochio, JN, JHEP 1502 (2015) 051



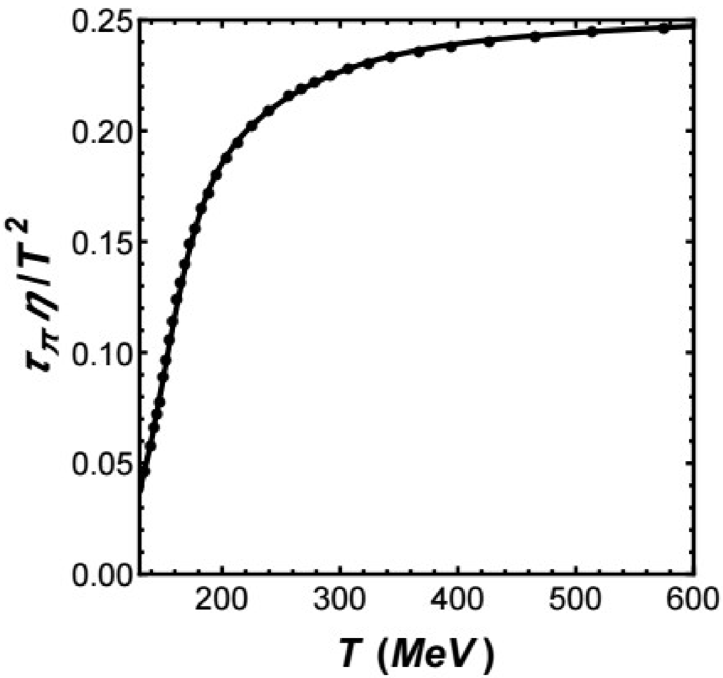
Small value for this transport coefficient in the QGP

2^{nd} order transport coefficients

The shear relaxation time

S. Finazzo, R. Rougemont, H. Marrochio, JN, JHEP 1502 (2015) 051

Universal formula in the bulk derived in



Obtained from a Kubo formula

$$\tau_\pi = \frac{1}{2\eta}\left(\lim_{q\to 0}\lim_{\omega\to 0}\frac{\partial^2 G_R^{xy,xy}(\omega,q)}{\partial\omega^2} - \kappa + T\frac{d\kappa}{dT}\right)$$

Parametrization for hydro

$$\tau_\pi \eta / T^2\left(x = \frac{T}{T_c}\right) = \frac{a}{1 + e^{b(c-x)} + e^{d(e-x)} + e^{f(g-x)}}$$

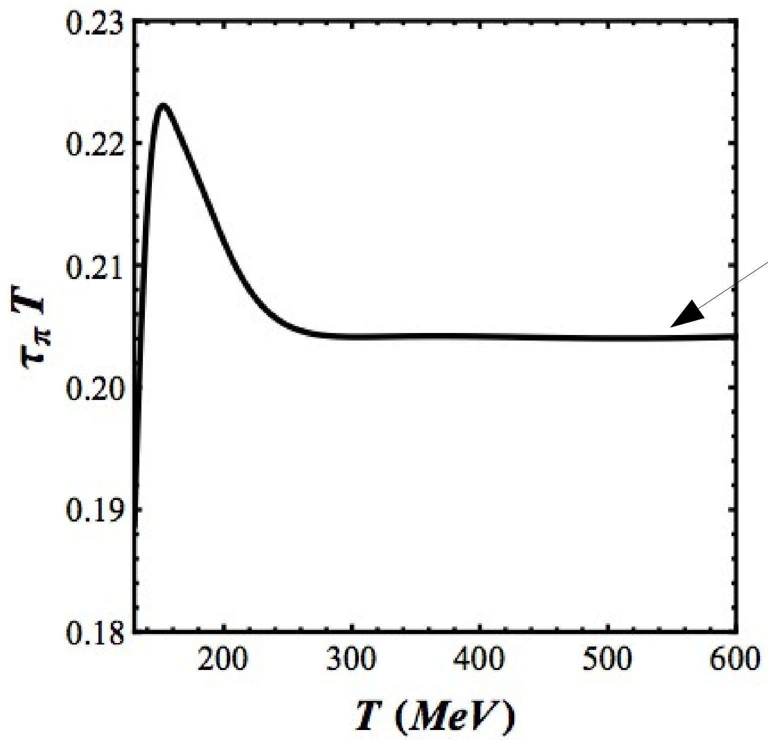
$T_c = 143.8$ MeV

a	b	c	d	e	f	g
0.2664	2.029	0.7413	0.1717	-10.76	9.763	1.074

2^{nd} order transport coefficients

The shear relaxation time

Shear relaxation time has a small peak in the region T ~ 150 – 250 MeV



CFT (SYM) value

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

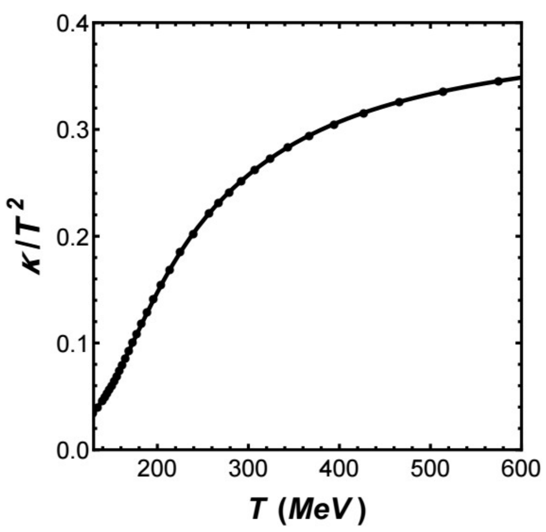
Very different than kinetic theory calculations!

$$\tau_\pi T \sim 2\,\eta/s$$

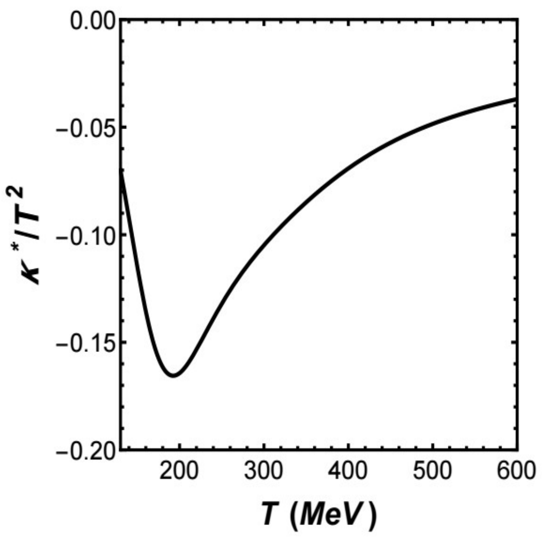
2^{nd} order transport coefficients

PREDICTIONS THAT CAN BE DIRECTLY TESTED ON THE LATTICE

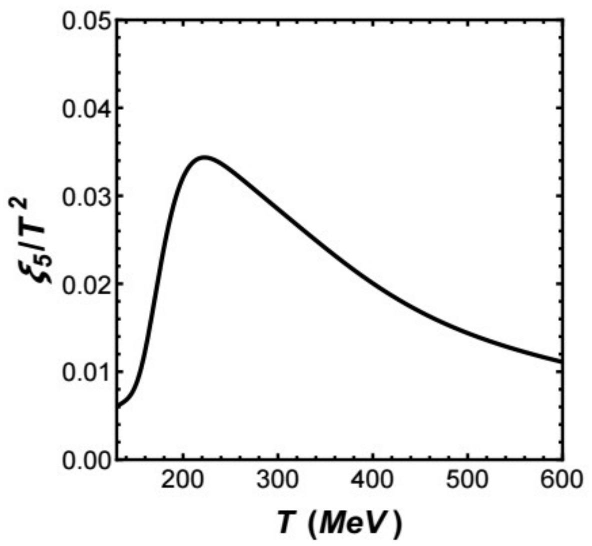
$$\kappa = -\lim_{q\to 0}\lim_{\omega\to 0}\frac{\partial^2 G_R^{xy,xy}(\omega,q)}{\partial q^2}$$



$$\kappa^* = \kappa - \frac{T}{2}\frac{d\kappa}{dT}$$



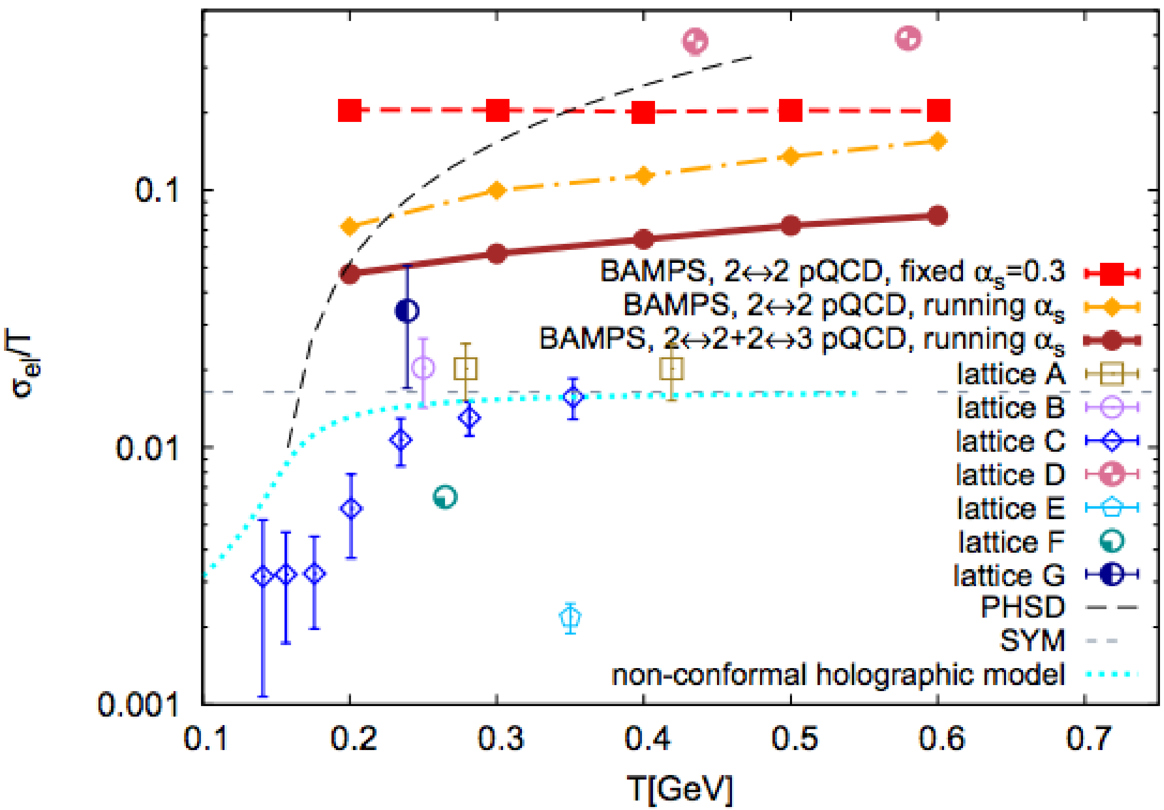
$$\xi_5 = \frac{1}{2}\left(c_s^2 T\frac{d\kappa}{dT} - c_s^2\kappa - \frac{\kappa}{3}\right)$$



Electric conductivity

(still at zero chemical potential)

S. I. Finazzo and J. Noronha, Phys. Rev. D 89, 106008 (2014).

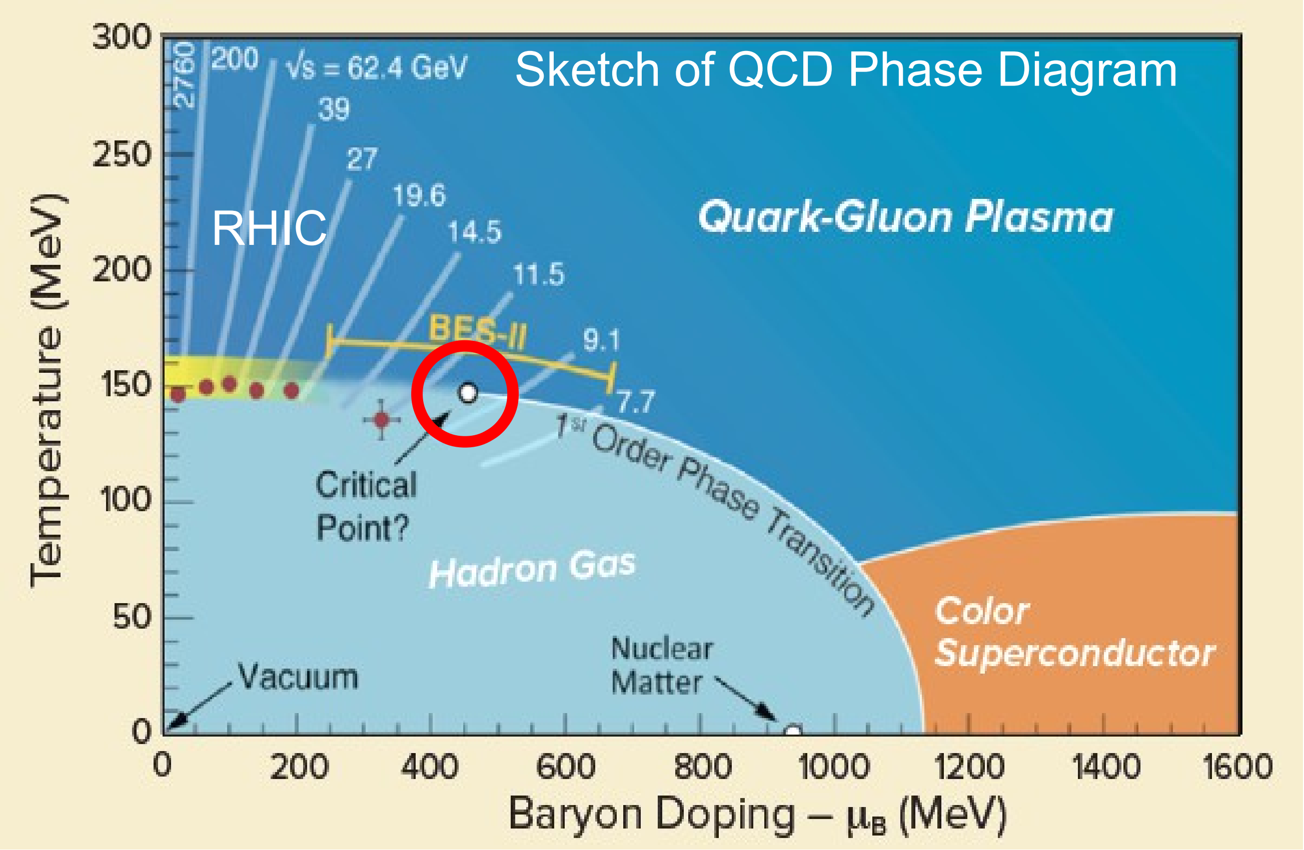


Model seems to be on the right track for thermodynamics and transport

“Doping” the holographic QGP with quarks

QCD Critical Point??

Perfect fluidity in a baryon-rich QGP???



2018 ???

"Doping" the holographic QGP with quarks

R. Rougemont, J. Noronha-Hostler, JN, PRL 2015

Effects from a conserved baryon charge $Q_B \to \mu_B \neq 0$

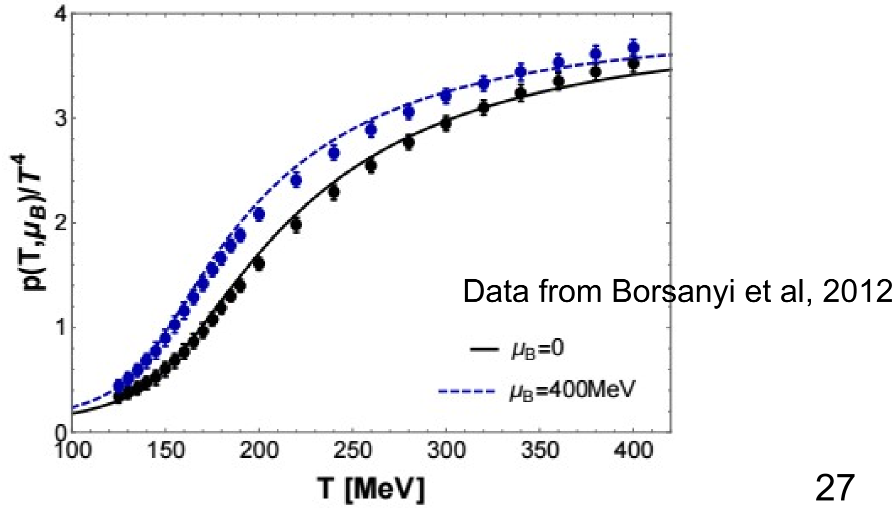
$$S = \frac{1}{16\pi G_5}\int d^5\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}(\partial_M\Phi)^2 - V(\Phi) - \frac{f(\phi)}{4}F_{MN}^2\right]$$

$$f(\phi) = \frac{\mathrm{sech}(1.2\,\phi - 0.69)}{3\,\mathrm{sech}(0.69)} + \frac{2e^{-100\,\phi}}{3}$$

Fixed by baryon susceptibility

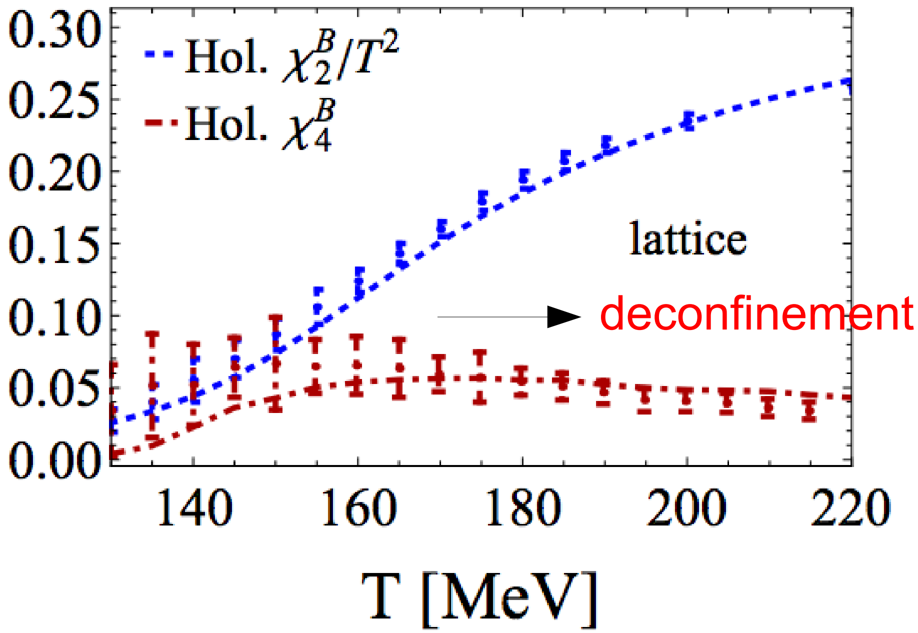
$$\chi_2^B(T) = \frac{\partial\rho_B}{\partial\mu_B}\bigg|_{\mu_B=0}$$

Describes lattice data

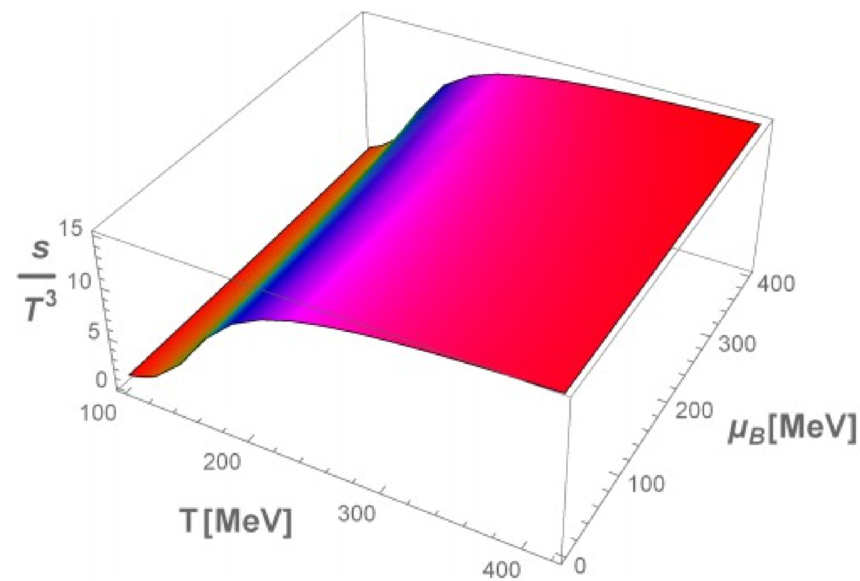


“Doping” the holographic QGP with quarks

R. Rougemont, J. Noronha-Hostler, JN, PRL 2015.



100000 black brane configurations



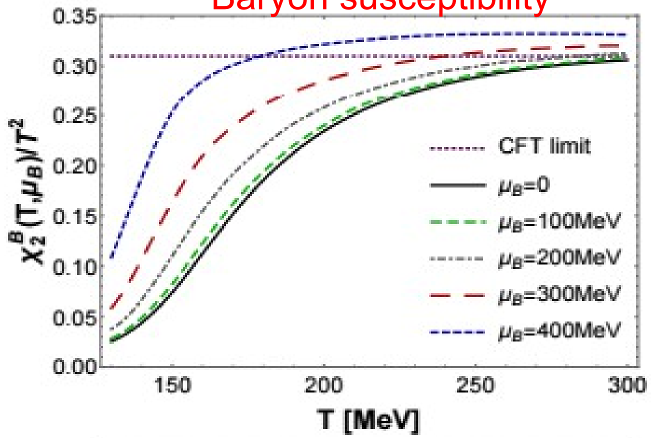
Model describes baryon charge effects at transition (other models?)

“Doping” the holographic QGP with quarks

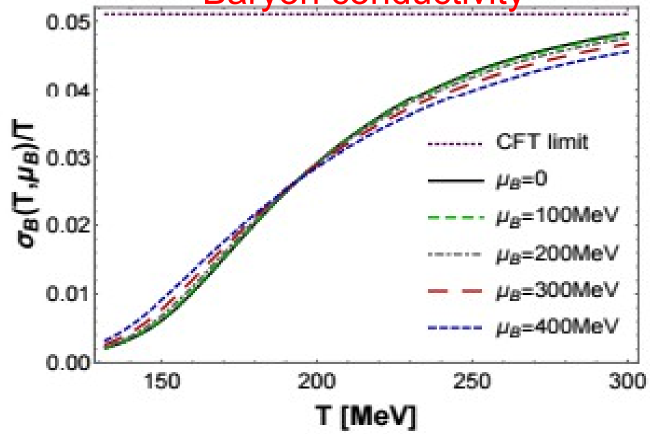
R. Rougemont, J. Noronha-Hostler, JN, PRL 2015.

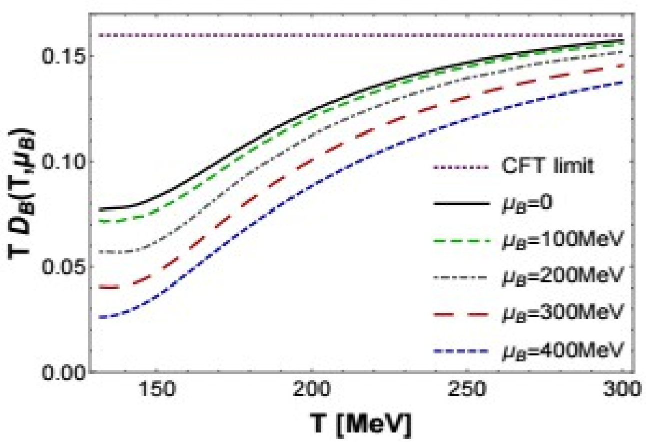
Suppression of baryon diffusion and transport for collisions in the BES regime

Baryon susceptibility

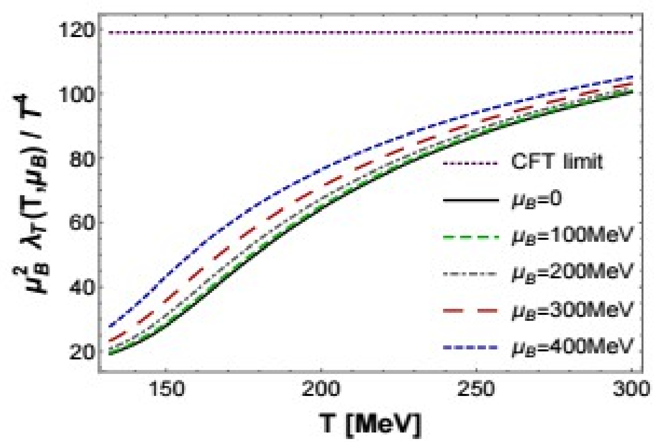


Baryon conductivity





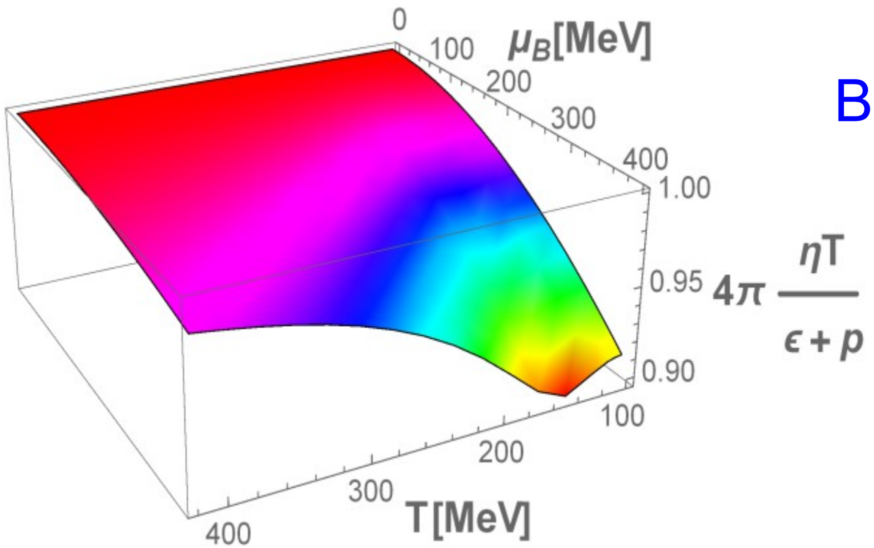
Baryon diffusion



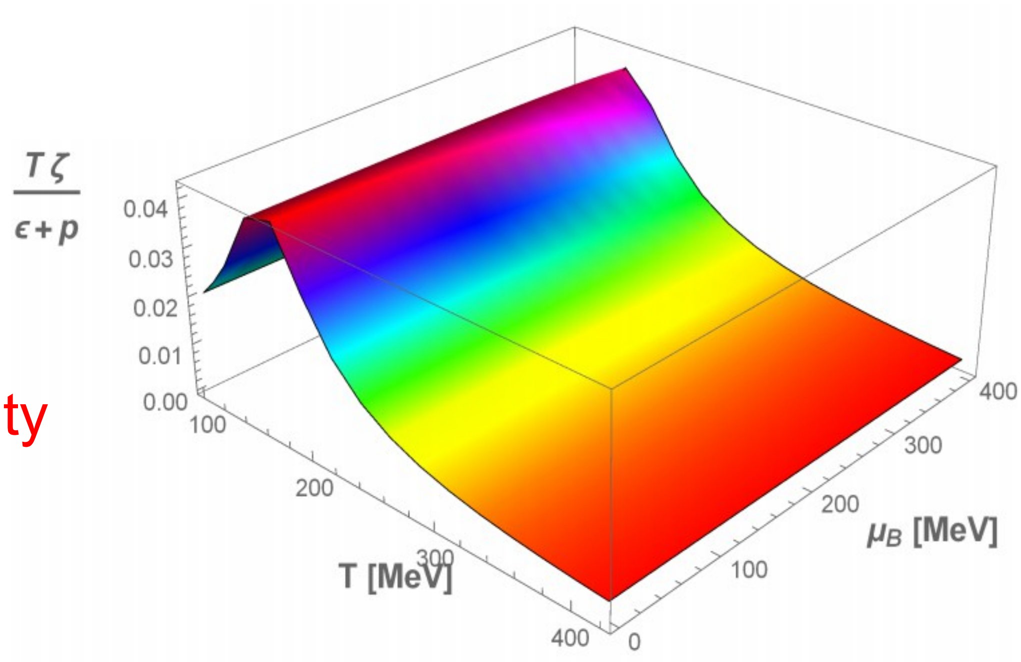
Thermal conductivity

"Doping" the holographic QGP with quarks

R. Rougemont, A. Ficnar, S. Finazzo, R. Critelli, J. Noronha-Hostler, JN, to appear soon



Baryon rich QGP is a perfect fluid

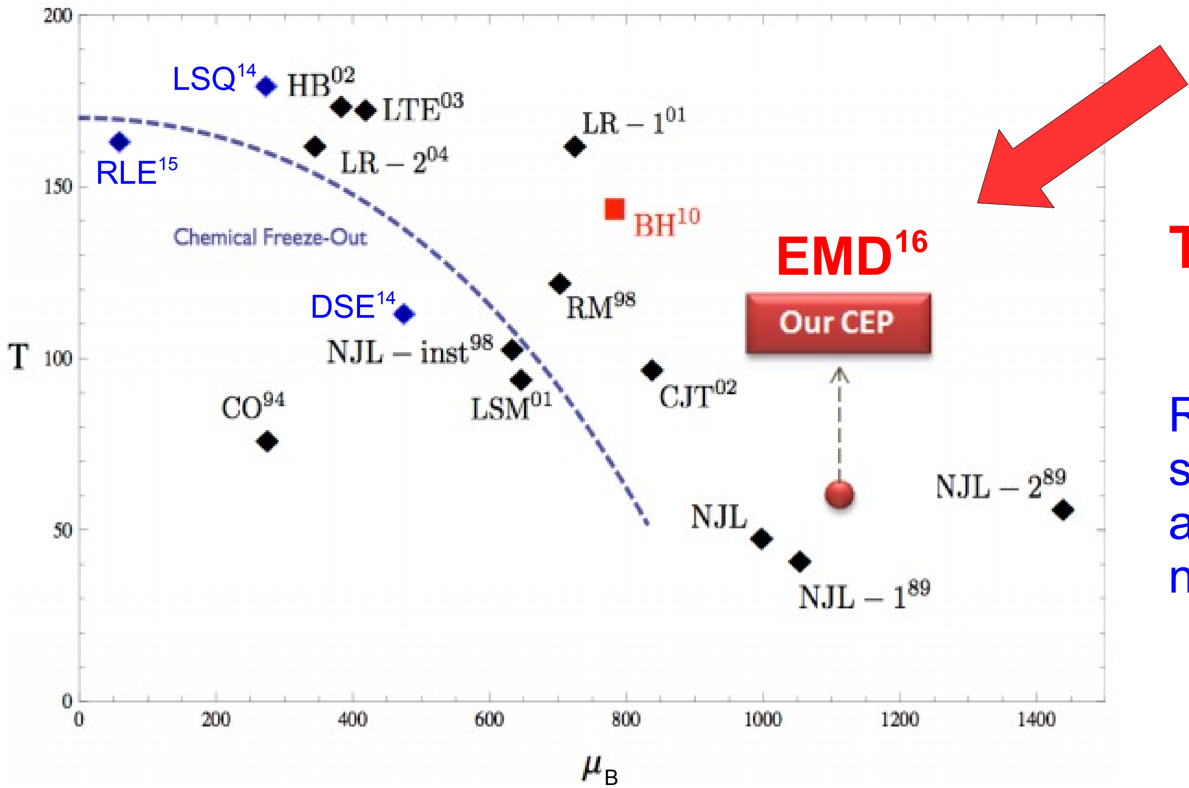


Small bulk viscosity

Prediction for the critical endpoint

R. Rougemont, A. Ficnar, S. Finazzo, R. Critelli, J. Noronha-Hostler, JN, to appear soon

We find $(T^{\rm CEP}, \mu_B^{\rm CEP}) = (65.6, 1123.6)\,\mathrm{MeV}$



Too far out for BES

Right ballpark of for some astrophysical applications (e.g., proto neutron stars)

(Original plot from DeWolfe, Gubser, Rosen, PRD circa 2010)

Final Remarks

- Black hole engineering provides a tool for computing a large number (~18 so far) transport coefficients of a strongly coupled non-conformal QGP.

- Transport coefficients + EOS (which includes CEP) are available for use in hydrodynamic simulations.

- At strong coupling, bulk viscosity appears to be small. Relaxation time also small.

- Baryon rich QGP still a perfect fluid. CEP out of reach of BES.

- Many ways to improve and extend current calculations (Nc corrections + finite coupling + D-brane dynamics)

EXTRA SLIDES

<u>Holography becomes simple when:</u>

I) The coupling of the QFT, say, λ , is $\lambda \gg 1$

II) The number of d.o.f./volume, N, is very large, i.e., N >> 1.

$\lambda \gg 1$

4-dimensional QFT
with d.of./vol $\to \infty$

HOLOGRAPHY

$\sim 1/\lambda \ll 1$

5-dimensional
classical gravity
+ other fields

- Applications in different systems ranging from particle physics to condensed matter physics.

Non-conformal relativistic hydrodynamics at 2^{nd} order in gradients

Note the increase in the number of T-dependent coefficients:

- 0^{th} order (ideal fluid): $c_s^2 = dP/d\varepsilon$
- 1^{st} order (Navier-Stokes): $c_s^2 = dP/d\varepsilon$, η and ζ
- 2^{nd} order theory: $c_s^2 = dP/d\varepsilon$

(can be computed on the lattice)

$\kappa, \kappa^*, \lambda_3, \lambda_4, \xi_3, \xi_4, \xi_5, \xi_6$ ⟶ Determined via **Euclidean** 2 and 3-point functions

$\eta, \zeta, \tau_\pi, \tau_\pi^*, \tau_\Pi, \lambda_1, \lambda_2, \xi_1,$ and ξ_2 ⟶ Dissipative properties (**imaginary parts** of retarded 2 and 3-point functions)

Note that $\kappa, \kappa^*, \xi_5,$ and ξ_6 do not contribute to EOM in flat spacetime.

Non-conformal relativistic hydrodynamics at 2nd order in gradients

- Even in flat spacetime, there are still 13 temperature dependent transport coefficients to be computed.

- At 2nd order, qualitatively new terms appear involving

Vorticity	Vorticity + shear coupling	Shear + bulk coupling terms	
$\lambda_3 \Omega^{\langle\mu}_{\lambda} \Omega^{\nu\rangle\lambda}$ $\xi_3 \Omega_{\mu\nu} \Omega^{\mu\nu}$	$\lambda_2 \sigma^{\langle\mu}_{\lambda} \Omega^{\nu\rangle\lambda}$	$\eta \tau^*_\pi \sigma^{\mu\nu} \dfrac{\theta}{3}$	$\xi_1 \sigma_{\mu\nu} \sigma^{\mu\nu}$

- Now, shear and bulk channels interact directly via EOM.

- Conformal invariance at early stages is broken by time evolution.

- Temperature dependent transport coefficients are predictions of the model.

Once the speed of sound is fixed (i.e., the EOS) the transport coefficients are completely defined by the equilibrium properties of the black brane + holography.

- Our results for the transport coefficients provide an answer for the case of a holographic non-conformal plasma with equilibrium properties similar to QCD near crossover.

- All the 13 transport coefficients for the non-conformal QGP that appear at this order were determined in JHEP 1502 (2015) 051.

Shear viscosity

S. Finazzo, R. Rougemont, H. Marrochio, JN, JHEP 1502 (2015) 051

Universality of isotropic black brane horizons (KSS PRL 2005)

$$\boxed{\eta/s = 1/(4\pi)}$$

Kubo formula

$$\eta = -\lim_{q\to 0}\lim_{\omega\to 0} \mathrm{Im}\left[\frac{\partial G_R^{xy,xy}(\omega,q)}{\partial\omega}\right]$$

$$G_R^{xy,xy}(\omega,\vec{q}) = -i\int_{\mathbb{R}^{1,3}} d^4x\, e^{i(\omega t-\vec{q}\cdot\vec{x})}\,\theta(t)\langle[\hat{T}^{xy}(t,\vec{x}),\hat{T}^{xy}(0,\vec{0})]\rangle$$

- Value in the correct ballpark for heavy ions.

- This fails away from “the Goldilocks temperature zone”

“Doping” the holographic QGP with quarks

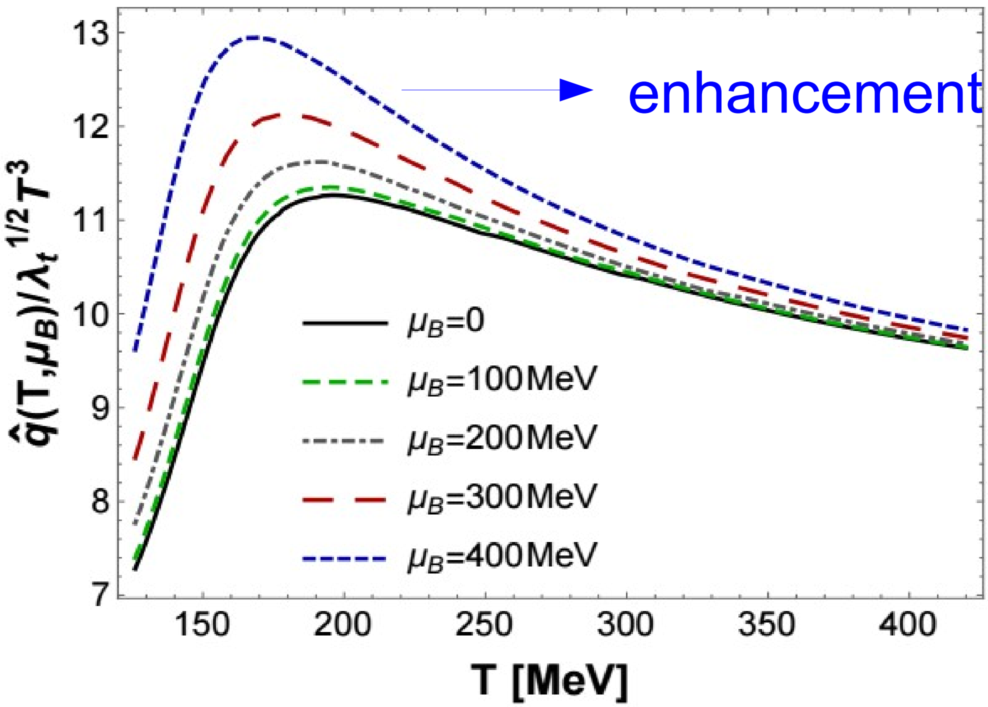
R. Rougemont, A. Ficnar, S. Finazzo, JN, arXiv:1507.06556 [hep-th] (JHEP).

Predictions for light quark energy loss

$$\hat{q} \equiv -\frac{4\sqrt{2}}{L^{-}L^{2}} \times \ln\left(\langle W^{(\mathrm{adjoint})}_{L\times L^{-}}\rangle\right)$$

LRW, 2006

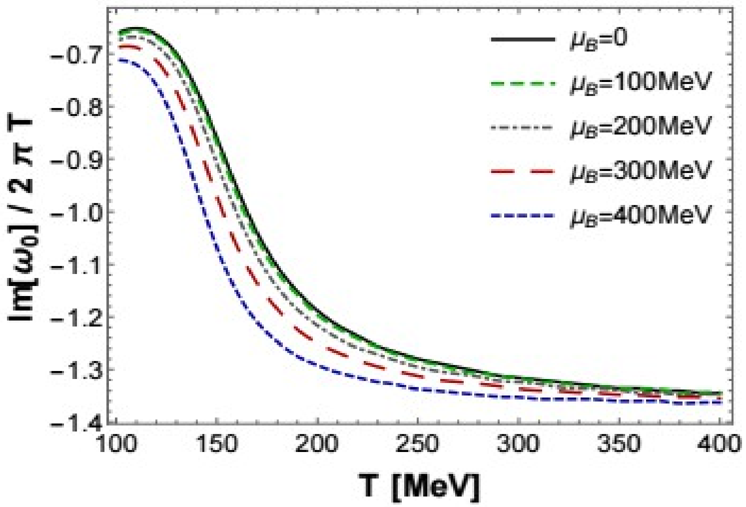
Jet quenching parameter

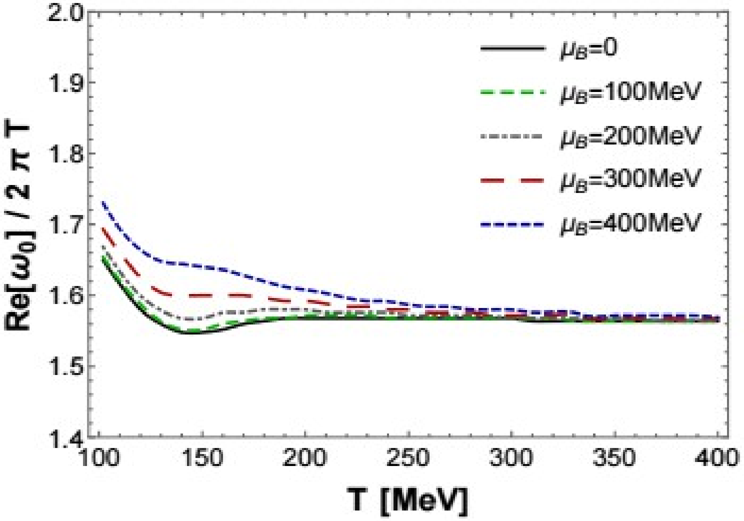


enhancement!!!

Quasinormal Ringdown of the QGP

R. Rougemont, A. Ficnar, S. Finazzo, JN, arXiv:1507.06556 [hep-th] (JHEP).





Standard hydrodynamics appears (after QNM decay)

Time/length scale $\sim 0.2/T \rightarrow$ sub-nucleon scales

There is no effective theory at the moment that can describe such oscillations $\rightarrow$ Effects on QGP phenomenology unknown

A new type of phase transition

A. Ficnar, JN, to appear soon

Relativistic fluids have a well defined Navier-Stokes limit

$$\pi_{NS}^{\mu\nu} = -\eta\sigma^{\mu\nu}$$

But how is this asymptotic limit reached (dynamically)?

Weak coupling → quasiparticles → Transport (Boltzmann-like)

$$p^\mu \partial_\mu f = \mathcal{C}[f] \implies \tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \dots$$

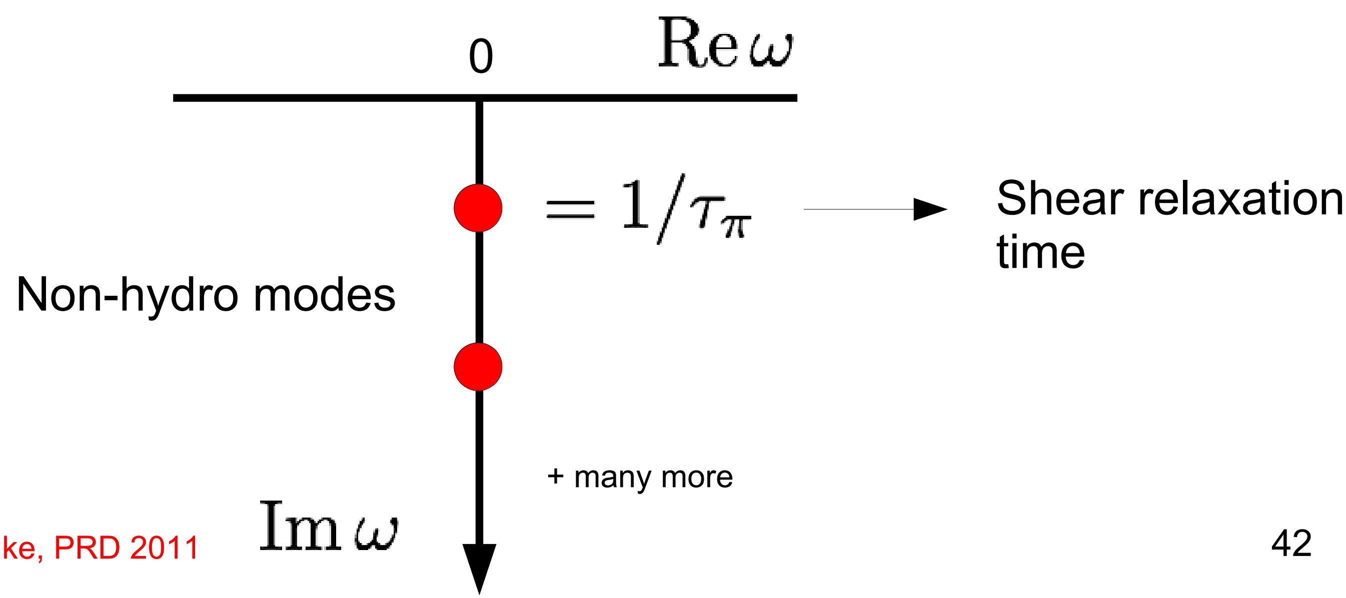
G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

Weak coupling → quasiparticles → Transport (Boltzmann-like)

Linear level $$\delta\pi^{\mu\nu}(\omega) = G_R^{xy,\mu\nu}(\omega)\delta\sigma_{\mu\nu}(\omega)$$

$G_R^{xy,\mu\nu}(\omega)$ → meromorphic function with purely imaginary poles

Navier-Stokes limit → decay of non-hydrodynamic modes



G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

Strong coupling → Holography → Quasinormal modes (QNM)

On-shell gravity action → generator of retarded correlators

Son, Starinets, 2002

$$\varphi(z) \equiv \delta h_y^x(z) \quad \Rightarrow \quad \Box\varphi = 0 \quad \Rightarrow \quad G_R^{xy,\mu\nu}(\omega)$$

Massless scalar field coupled to gravity in the bulk

Retarded correlator in the gauge theory

$z = 0$ $\varphi_0 = 0$

Dirichlet

Discrete QNM

ω_n

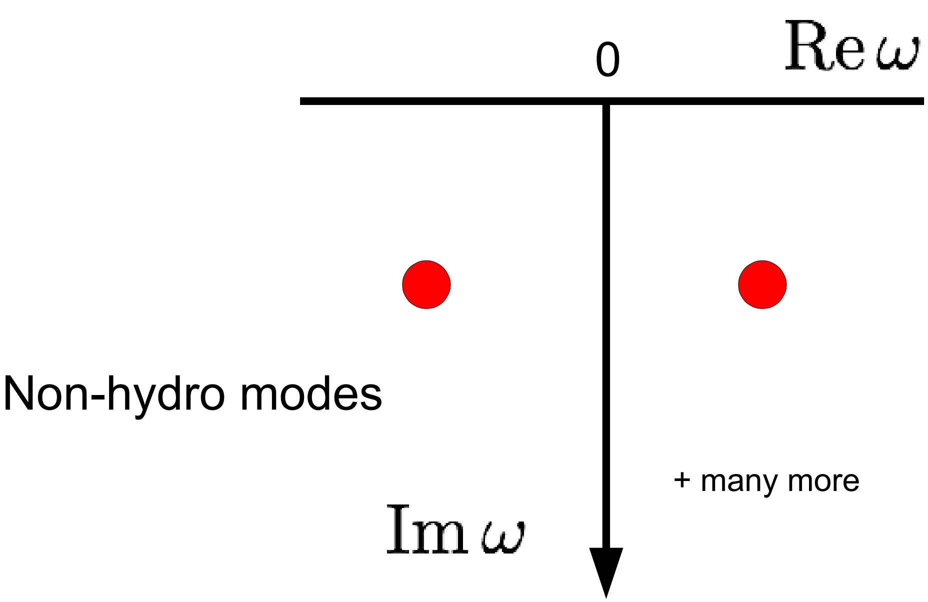
incoming wave at horizon

z_h

A new type of non-equilibrium phase transition

Computed holographically $$\delta\pi^{\mu\nu}(\omega) = G_R^{xy,\mu\nu}(\omega)\delta\sigma_{\mu\nu}(\omega)$$

Navier-Stokes limit $\rightarrow$ Decay + <u>Oscillation</u> of non-hydrodynamic modes

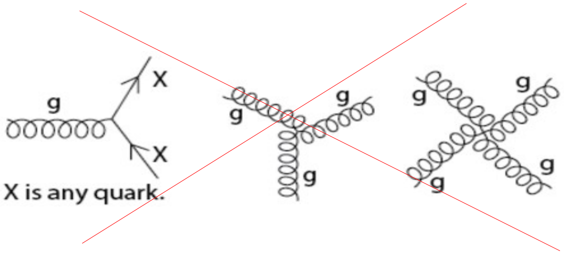


UNLIKE ANY FLUID

G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

$\mathrm{Re}\,\omega \rightarrow$ Order parameter

At strong coupling, a quasiparticle description is not useful



A new organizing principle is needed.

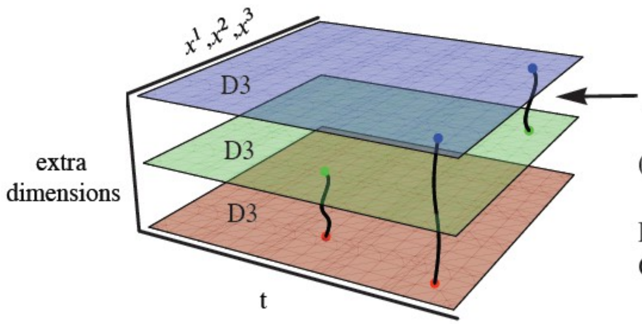
Perfect fluidity should naturally follow directly from it.

Holography is the only approach where this occurs

Once you eliminate the impossible, whatever remains, no matter how improbable, must be the truth.

STANDARD EXAMPLE

$\mathcal{N} = 4$ SU(Nc) Supersymmetric Yang-Mills in d=4



Fields in the adjoint rep. of SU(Nc)

- 16 + 16 supercharges
- SU(4) R-symmetry
- SO(6) global symmetry

$\beta = 0$ **CFT !!!!**

Maldacena, 1997: This gauge theory is dual to Type IIB string theory on AdS_5 x S_5

Strongly-coupled, large Nc gauge theory	Weakly-coupled, low energy string theory
$N_c \to \infty$	$g_s \to 0$
$\lambda = R^4/\ell_s^4 \to \infty$	$\ell_s/R \to 0$
t'Hooft coupling in the gauge theory	

Universality and perfect fluidity

$\lambda \gg 1$ in QFT $\rightarrow$ string theory in weakly curved backgrounds

d.o.f. / vol. $\rightarrow \infty$ in QFT $\rightarrow$ vanishing string coupling

T, μ in QFT $\rightarrow$ spatially isotropic black brane

The most general theory in the bulk is:

A theory of gravity (+ other fields) with at most 2 derivatives

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\,(R + \underbrace{\Lambda}_{\text{negative}} + \text{other fields})$$

On-shell gravity action → generator of retarded correlators

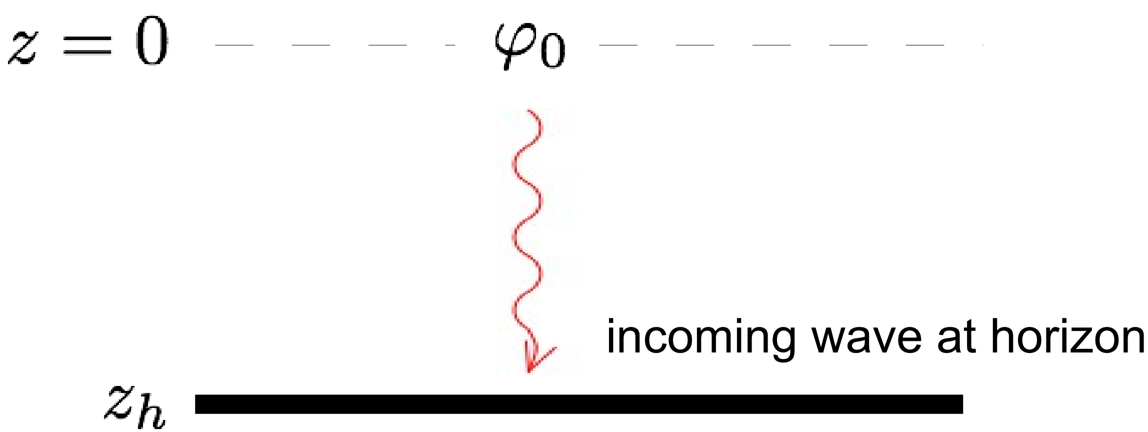
Son, Starinets, 2002

Linearizing the action $g_{MN} \to g_{MN} + \delta h_{MN}$

$$\varphi(z) \equiv \delta h^x_y(z) \quad \Longrightarrow \quad \Box\varphi = 0 \quad \Longrightarrow \quad G_R^{xy,xy}$$

Massless scalar field coupled to gravity in the bulk

Retarded correlator in the gauge theory



Entropy density $\rightarrow$ $s = \frac{\text{area}}{4G_5}$ Bekenstein's area law

$$\eta = i\,\partial_\omega G_R^{xyxy}(\omega, \mathbf{0})\Big|_{\omega=0} = \frac{\text{area}}{16\pi G_5}$$

UNIVERSAL

$\sigma_{abs}(0) = \text{area}$

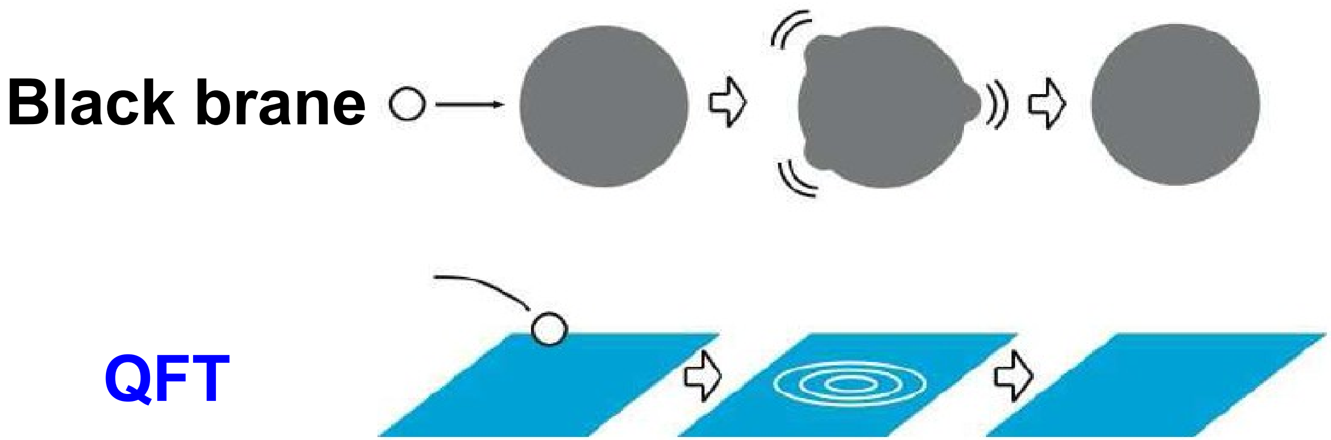
Das, Gibbons, Mathur, 1996

Kovtun,Son,Starinets, 2005

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

Universality of black hole horizons $\Rightarrow$ **HOLOGRAPHY** $\Rightarrow$ Universality of transport coefficient in QFT

Universality of black hole horizons → **HOLOGRAPHY** → Universality of transport coefficients in QFT



Dissipation of sound waves = Dissipation of black hole horizon disturbances

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

Results for the 2nd order transport coefficients:

coefficients λ_3 and λ_4

These coefficients are associated with the shear channel in the EOM

$$\boxed{\lambda_3 \Omega^{\langle\mu}_{\lambda}\Omega^{\nu\rangle\lambda}}$$

Require calculation of Euclidean 3-point function !!!!

(could be computed on the lattice)

$$\lambda_3 = 2\kappa^* - 4 \lim_{p_z, q_z \to 0} \frac{\partial^2}{\partial p_z \partial q_z} G_E^{xt,yt,xy}(p_t = 0, \vec{p}, q_t = 0, \vec{q})$$

$$\boxed{\lambda_4 \nabla^{\langle\mu} \ln s \, \nabla^{\nu\rangle} \ln s}$$

$$\lambda_4 = -2\kappa^* + \kappa - \frac{c_s^4}{2} \lim_{p_x, q_y \to 0} \frac{\partial^2}{\partial p_x \partial q_y} G_E^{tt,tt,xy}(p_t = 0, \vec{p}, q_t = 0, \vec{q})$$

Results for the 2nd order transport coefficients:

coefficients λ_3 and λ_4

- 3-point functions in non-conformal holography is a formidable task (work in progress).

- Our best estimate for these coefficients here consists in taking the CFT value for the 3-point functions while fully taking into account non-conformal effects in the other terms of the Kubo formulas that define them.

$$\lim_{p_z,q_z\to 0} \frac{\partial^2}{\partial p_z \partial q_z} G_E^{xt,yt,xy}(p_t=0,\vec{p},q_t=0,\vec{q}) = 0. \qquad \lim_{p_x,q_y\to 0} \frac{\partial^2}{\partial p_x \partial q_y} G_E^{tt,tt,xy}(p_t=0,\vec{p},q_t=0,\vec{q}) = \frac{2\kappa}{c_s^4}$$

$$\boxed{\lambda_3 = -\lambda_4 = 2\kappa^*}$$

Vorticity + shear coupling is not small !!!

Results for the 2nd order transport coefficients:

coefficients ξ_3, ξ_4

These coefficients are associated with the bulk channel in the EOM

G. D. Moore and K. A. Sohrabi, JHEP **11** (2012) 148

$$\xi_3 \Omega_{\mu\nu} \Omega^{\mu\nu}$$

(could be computed on the lattice)

$$\xi_3 = \frac{3c_s^2}{2} T \left(\frac{d\kappa^*}{dT} - \frac{d\kappa}{dT} \right) + \frac{3}{2}(c_s^2 - 1)(\kappa^* - \kappa) - \frac{\lambda_4}{c_s^2} + \frac{1}{4} \left(c_s^2 T \frac{d\lambda_3}{dT} - 3c_s^2 \lambda_3 + \frac{\lambda_3}{3} \right)$$

$$\xi_4 \nabla_\mu^\perp \ln s \, \nabla_\perp^\mu \ln s$$

(could be computed on the lattice)

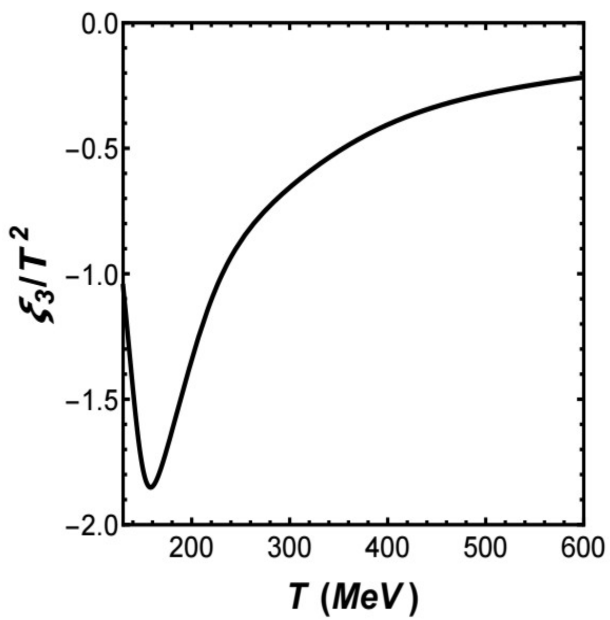
$$\xi_4 = -\frac{\lambda_4}{6} - \frac{c_s^2}{2} \left(\lambda_4 + T \frac{d\lambda_4}{dT} \right) + c_s^4 (1 - 3c_s^2) \left(T \frac{d\kappa}{dT} - T \frac{d\kappa^*}{dT} + \kappa^* - \kappa \right) + \\ - c_s^6 T^3 \frac{d^2}{dT^2} \left(\frac{\kappa - \kappa^*}{T} \right),$$

Results for the 2nd order transport coefficients:

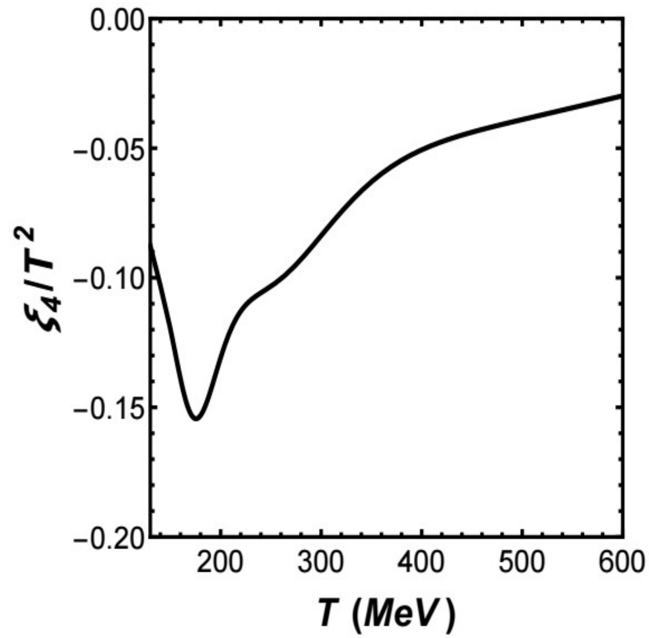
coefficients ξ_3, ξ_4

$$\xi_3 \Omega_{\mu\nu} \Omega^{\mu\nu}$$

$$\xi_4 \nabla^{\perp}_{\mu} \ln s \, \nabla^{\mu}_{\perp} \ln s$$



Vorticity effect: bulk channel



2nd order T gradient: bulk channel

Results for the 2nd order transport coefficients:

coefficient ξ_6

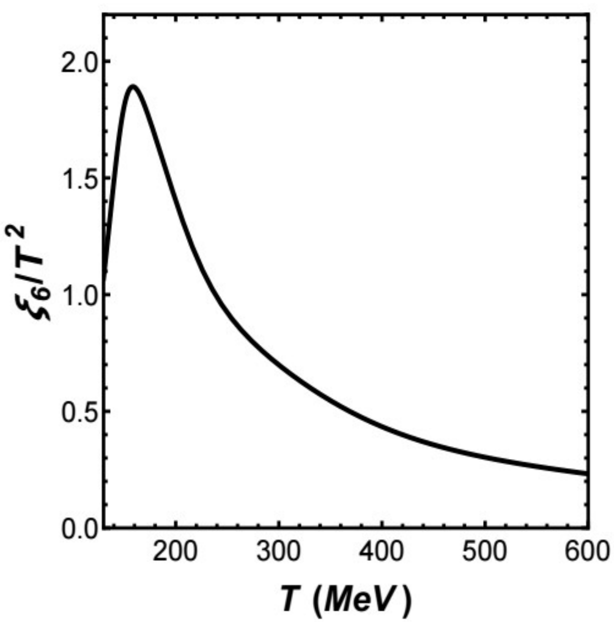
This coefficient is also associated with the bulk channel in the EOM

G. D. Moore and K. A. Sohrabi, JHEP **11** (2012) 148

$\boxed{\xi_6 u^\mu u^\nu \mathcal{R}_{\mu\nu}}$ Vanishes from EOM in flat space

(could be computed on the lattice)

$$\xi_6 = c_s^2\left(3T\frac{d\kappa}{dT} - 2T\frac{d\kappa^*}{dT} + 2\kappa^* - 3\kappa\right) - \kappa + \frac{4\kappa^*}{3} + \frac{\lambda_4}{c_s^2}$$



Note that ξ_3, ξ_4, ξ_6 are not small.

Vorticity+bulk coupling may be important !!!

Lattice QCD will play a role here !!!

Results for the 2nd order transport coefficients:

A lower bound estimate for τ_Π

While the calculation of the bulk relaxation time can be done using a method similar to the one we developed for the shear relaxation time, here we use:

Causality/linear stability constraint (valid for any Israel-Stewart-like theory)

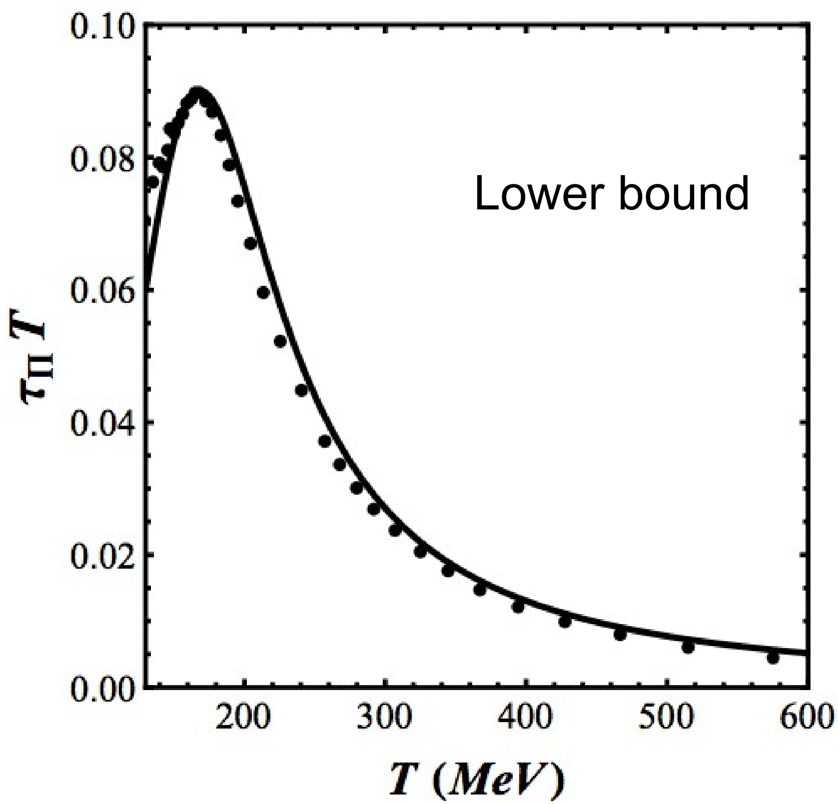
S. Pu, T. Koide and D. H. Rischke, Phys. Rev. D **81**, 114039 (2010)

$$\boxed{\frac{\zeta}{s\tau_\Pi T} + \frac{\eta}{s\tau_\pi T} \leq 1 - c_s^2}$$

This gives the smallest bulk relaxation time that this system can have.

Results for the 2nd order transport coefficients:

A lower bound estimate for τ_Π



Parametrization for hydro

$$\tau_\Pi T\left(x = \frac{T}{T_c}\right) = \frac{a}{\sqrt{(x-b)^2 + c^2}} + \frac{d}{x}$$

$T_c = 143.8$ MeV

a	b	c	d
0.05298	1.131	0.3958	-0.05060

Results for the 2nd order transport coefficients:

$$\lambda_1,\ \lambda_2,\ \xi_1,\ \xi_2,\ \text{and}\ \tau_\pi^*$$

- Very little is known about these coefficients ...

For strongly coupled SYM theory:

$$\lambda_1 = 2\frac{\eta^2}{sT}, \qquad \lambda_2 = -\ln 2\frac{\eta}{\pi T}$$

$$4\lambda_1 + \lambda_2 = 2\eta\tau_\pi$$

For the non-conformal plasma constructed via dimensional reduction of a higher dimensional pure gravity action one finds (this is OK for our model)

I. Kanitscheider and K. Skenderis, JHEP **0904**, 062 (2009)

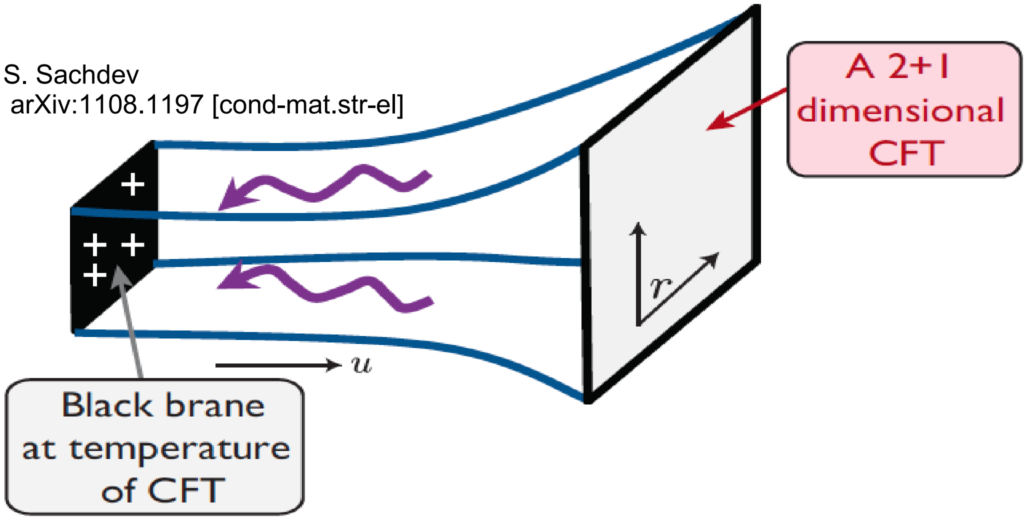
Shear + bulk coupling

$$\xi_1 = \lambda_1\left(\frac{1}{3} - c_s^2\right) \qquad \tau_\pi^* = -3\tau_\pi\left(\frac{1}{3} - c_s^2\right) \qquad \xi_2 = 2\eta\tau_\pi c_s^2\left(\frac{1}{3} - c_s^2\right)$$

Applications in condensed matter physics???

Rougemont, Noronha, Zarro, Guimaraes, Wotzasek, Granado, JHEP 1507 (2015) 070

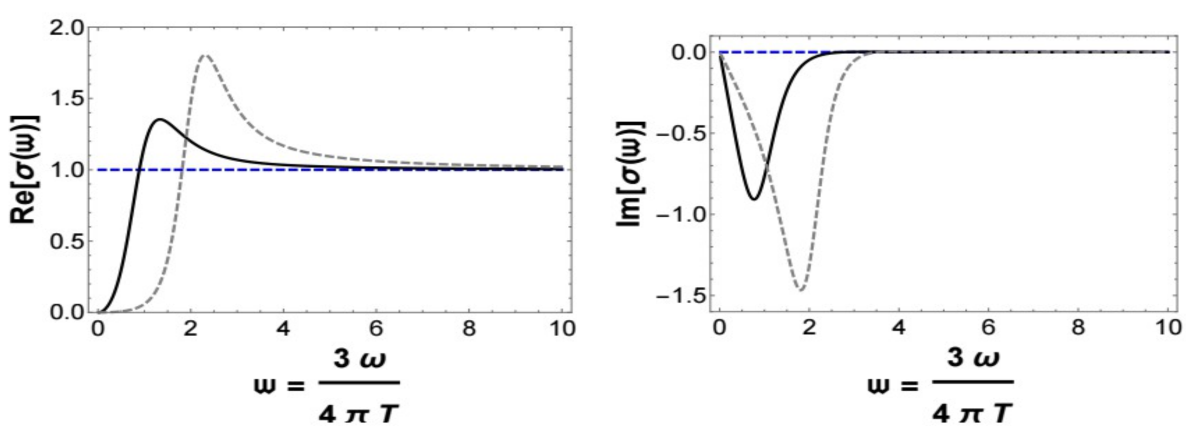
S. Sachdev
arXiv:1108.1197 [cond-mat.str-el]



+ Magnetic monopole condensate in the bulk

+ Perfect fluidity

Zero DC conductivity in a strongly interacting system on the plane



Non-conformal relativistic hydrodynamics at 2nd order in gradients

- This 2nd order gradient expansion theory, however, is still **acausal and unstable.**

S. Finazzo, R. Rougemont, H. Marrochio, JN, JHEP 1502 (2015) 051

- Common "trick" is to employ a type of Israel-Stewart-like resummation:

Use lowest order relations: $\sigma^{\mu\nu} \to -\pi^{\mu\nu}/\eta$ and $\theta \to -\Pi/\zeta$

To promote $\pi^{\mu\nu}$ and Π to independent dynamical variables that obey relaxation-like equations of motion.

Israel-Stewart-like, 2nd order hydrodynamic equations

This gives in this case (in flat spacetime):

S. Finazzo, R. Rougemont, H. Marrochio, JN, JHEP 1502 (2015) 051

Shear channel:

$$\tau_\pi \left(D\pi^{\langle\mu\nu\rangle} + \frac{4\theta}{3}\pi^{\mu\nu} \right) + \pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \frac{\lambda_1}{\eta^2}\pi_\lambda^{\langle\mu}\pi^{\nu\rangle\lambda} - \frac{\lambda_2}{\eta}\pi_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda} - \lambda_3\Omega_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda}$$
$$+\tau_\pi\,\pi^{\mu\nu}\,D\,\ln\left(\frac{\eta}{s}\right) + \tau_\pi^*\pi^{\mu\nu}\frac{\Pi}{3\zeta} + \lambda_4\nabla^{\langle\mu}\ln s\,\nabla^{\nu\rangle}\ln s$$

Bulk channel: $$\tau_\Pi\,(D\Pi + \Pi\theta) + \Pi = -\zeta\theta + \frac{\xi_1}{\eta^2}\pi_{\mu\nu}\pi^{\mu\nu} + \frac{\xi_2}{\zeta^2}\Pi^2 + \xi_3\Omega_{\mu\nu}\Omega^{\mu\nu}$$
$$+\tau_\Pi\,\Pi\,D\,\ln\left(\frac{\zeta}{s}\right) + \xi_4\nabla_\mu^\perp\ln s\,\nabla_\perp^\mu\ln s\,.$$

These equations can be **causal and linearly stable**. They are being implemented in current numerical hydro codes.

Israel-Stewart-like, 2nd order hydrodynamic equations

- "UV completion" preserves the number of transport coefficients found in the gradient expansion.

- Asymptotic behavior is guaranteed to coincide with 2nd order gradient expansion.

- Shear and bulk channels after resummation obey their own differential equations.

- Clearly, this particular UV completion (which leads to relaxation equations for the dissipative currents) **is not unique**.

Ex: $\ddot{x} + \gamma\dot{x} + x = f(t)$ and $\gamma\dot{x} + x = f(t) \xrightarrow{\text{same}} x_{asymp}(t) \sim f(t)$ + ...